Workshop on Nonlinear Partial Differential Equations -China-Japan Joint Project for Young Mathematicians 2016

November-14-16, 2016
Venue: Room 510, Science Building, Zhongshan Road Campus,
East China Normal University, Shanghai, P.R. China
http://www.math.ecnu.edu.cn/events.html

Program

November 14 (Monday)

9:25 – 9:30 **Opening**: F. Zhou (East China Normal University)

– Chair: Y. Morita –

9:30 – 10:15 **Y. Sugiyama** (Kyushu University)
"Time Global Existence and Finite Time Blow-up Criterion for Solutions to the Keller-Segel System Coupled with Navier-Stokes Fluid"

10:20 – 11:05 **L. Zhang** (Jiaotong University)
"Uniqueness and Traveling Waves in a Cell Motility Model"

Tea Break

11:25 – 12:10 **K. Takasao** (University of Tokyo)
"Global Existence of Weak Solutions for Volume Preserving Mean Curvature Flow"

Lunch Break

– Chair: F. Zhou –

14:00 – 14:45 **B. Lou** (Shanghai Normal University)
"The Fisher-KPP Equation with Free Boundaries in Time-Periodic Advective Environment"

Tea Break

15:05 – 15:50 **N. Ikoma** (Kanazawa University)
"Existence of Solutions for Scalar Field Equations Involving Fractional Operators"

– Chair: L. Zhang –

15:55 – 16:20 **X.-Q. He** (East China Normal University)
"Spatial Heterogeneity and Time Periodicity in Lotka-Volterra Competition-Diffusion Systems"

16:20 – 16:45 **H.-C. Li** (East China Normal University)
"Qualitative Analysis on a Diffusive SIS Epidemic Model with Varying Total Population"

17:00 – 18:00 **Move to party place by bus**

18:00 – 20:00 **Party**

November 15 (Tuesday)

– Chair: B. Lou –

9:30 – 10:15 **Y. Fujishima** (Shizuoka University)
"Existence and Nonexistence of Solutions for the Heat Equation with a Superlinear Source Term"

10:20 – 11:05 **F.P. Yao** (Shanghai University)
"Higher Integrability for Nonlinear Parabolic equations of p-Laplacian Type

Tea Break

11:25 – 12:10 **X.-F. Xiang** (Tongji University)
"On the Shape of Meissner Solutions to a Limiting Form of Ginzburg-Landau Systems"

Lunch Break

– Chair: Y. Sugiyama –

14:00 – 14:45 **T. Wakasa** (Kyushu Institute of Techlonogy)
"Traveling Waves in a PDE Model of Tumour Growth"

Tea Break

15:05 – 15:50 **H.-Y. Chen** (Jiangxi Normal University)
"Isolated Singularities of Choquard Equations"

– Chair: Y. Fujishima –

15:55 – 16:20 **X. Huang** (East China Normal University)
"Conformal Metrics in $\mathbb{R}^{2m}$ with Constant Q-Curvature and Arbitrary Volume"

16:20 – 16:45 **Y. Zeng** (East China Normal University)
"Steady States of Hall-MHD System"

16:45 – 17:10 **Z.-B. Zhang** (East China Normal University)
"Existence and Regularity of Weak Solutions to a Thermoelectrical Model"

17:10 – 17:35 **Free discussions**

17:35 – 17:40 **Closing**: Y. Morita (Ryukoku University)

November 16 (Wednesday)

Free discussions

Organizers:
Yoshihisa Morita (Ryukoku Center for Mathematical Sciences, Ryukoku University)
Xingbin Pan (East China Normal University)
Feng Zhou (East China Normal University)

WORKSHOP ON NONLINEAR PARTIAL DIFFERENTIAL EQUATIONS : CHINA-JAPAN JOINT PROJECT FOR YOUNG MATHEMATICIANS 2016

November 14-16, 2016.

East China Normal University, Shanghai, China

Invited Lectures

Isolated Singularities of Choquard Equations

Huyuan Chen

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In this talk, we will discuss isolated singular solutions of Choquard equation

$$\begin{cases} -\Delta u + u = I_\alpha[u^p]u^q \quad \text{in} \quad \mathbb{R}^N \setminus \{0\}, \\ \lim\limits_{|x|\to+\infty} u(x) = 0, \end{cases} \tag{0.1}$$

where $p, q > 0$, $N \geq 3$, $\alpha \in (0, N)$ and

$$I_\alpha[u^p](x) = \int_{\mathbb{R}^N} \frac{u(y)^p}{|x-y|^{N-\alpha}}\, dy.$$

Isolated singularises of problem (0.1) is studied by building the connection with the very weak solution of

$$-\Delta u + u = I_\alpha[u^p]u^q + k\delta_0 \quad \text{in} \quad \mathbb{R}^N, \tag{0.2}$$

where δ_0 is the Dirac mass at the origin. This is joint work with Feng Zhou.

Existence and Nonexistence of Solutions for the Heat Equation with a Superlinear Source Term

Yohei Fujishima

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Classification theory on the existence and non-existence of local in time solutions for initial value problems of nonlinear heat equations are investigated. Without assuming a concrete growth rate on a nonlinear term, we reveal the threshold integrability of initial data which classify existence and nonexistence of solutions via a quasi-scaling and its invariant integral. Typical nonlinear terms, for instance polynomial type, exponential type and its sum, product and composition, can be treated as applications. This talk is based on a joint work with Professor Norisuke Ioku (Ehime University).

Higher Integrability for Nonlinear Parabolic Equations of p-Laplacian Type

Fengping Yao

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In this paper we give a new alternative proof of the local higher integrability in Sobolev spaces of the gradient for weak solutions of quasilinear parabolic equations of p-Laplacian type. Moreover, we would like to point out that our results are homogeneous regularity estimates and improve the known results for such equations by using some new techniques. Actually, our results can be extended to the global estimates and cover a more general class of degenerate/singular parabolic problems of p-Laplacian type.

Existence of Solutions for Scalar Field Equations Involving Fractional Operators

Norihisa Ikoma

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This talk is devoted to the existence of solutions

$$(-\Delta+1)^{\alpha}u = f(x,u) \quad \text{in } \mathbf{R}^N, \quad u \in H^{\alpha}(\mathbf{R}^N).$$

Here $0<\alpha<1$ and $(-\Delta+1)^{\alpha}$ is defined through the Fourier transform:

$$(-\Delta+1)^{\alpha}u := \mathcal{F}^{-1}\left(\left(1+4\pi^2|\xi|^2\right)(\mathcal{F}u)(\xi)\right), \quad (\mathcal{F}u)(\xi) := \int_{\mathbf{R}^N} e^{-2\pi i x\cdot\xi}u(x)dx.$$

We first treat the case $f(x,s) = f(s)$ and under general assumptions on $f(s)$ (which is called a Berestycki-Lions type nonlinearity), we show the existence of positive solution and its properties. Exploiting the properties we showed, the existence of positive solution is proved for the case $f(x,s)$.

The Fisher-KPP Equation with Free Boundaries in Time-Periodic Advective Environment

Bendong Lou

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In this talk we consider the Fisher-KPP equation with free boundaries in an environment with time-periodic advection:

$$\begin{cases} u_t = u_{xx} - \beta(t)u_x + f(t,u), & x \in (g(t), h(t)),\ t > 0, \\ u(t,g(t)) = 0,\ g'(t) = -\mu(t)u_x(t,g(t)), & t > 0, \\ u(t,h(t)) = 0,\ h'(t) = -\mu(t)u_x(t,h(t)), & t > 0, \\ g(0) = g_0, h(0) = h_0,\ u(0,x) = u_0(x), & x \in [g_0, h_0], \end{cases}$$

where β, μ and $f(\cdot,u)$ are all periodic in t, and $f(t,\cdot)$ is a Fisher-KPP type of nonlinearity. I will show the influence of $\beta(t)$ (in the presence of free boundaries) on the long time behavior of the solutions.

Time Global Existence and Finite Time Blow-up Criterion for Solutions to the Keller-Segel System Coupled with Navier-Stokes Fluid

Yoshie Sugiyama

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We will deal with the chemotaxis model under the effect of the Navier-Stokes fluid, *i.e.*, the incompressible viscous fluid. We shall show the existence of a local *mild solution* for large initial data and a global *mild solution* for small initial data in the scale invariant class demonstrating that $n_0 \in L^1(\mathbb{R}^2)$ and $u_0 \in L^2_\sigma(\mathbb{R}^2)$. Our method is based on the perturbation of linearization together with the $L^p - L^q$-estimates of the heat semigroup and the fractional powers of the Laplace operator. As a by-product of our method, we shall construct a self-similar solution and prove the smoothing effect and uniqueness of

our *mild solution*. In addition, we shall show a blow-up criterion which almost covers the well-known threshold number 8π of the size $\|n_0\|_{L^1(\mathbb{R}^2)}$ under the rest state of the fluid motion. Furthermore, the blow-up rate will be also discussed. This is based on a joint work with Professor Hideo KOZONO (Waseda university) and Mr. Masanari MIURA (Kyushu university).

Global Existence of Weak Solutions for Volume Preserving Mean Curvature Flow

Keisuke Takasao

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Let $U_t \subset \mathbb{R}^n$ be a bounded open set and have a smooth boundary M_t for $t \in [0,T)$. The family of hypersurfaces $\{M_t\}_{t\in[0,T)}$ is called the volume preserving mean curvature flow if the velocity vector v of M_t is given by

$$v = h - \Big(\frac{1}{\mathcal{H}^{n-1}(M_t)} \int_{M_t} h \cdot \nu \, d\mathcal{H}^{n-1}\Big)\nu \quad \text{on } M_t, \tag{0.3}$$

where h and ν are the mean curvature vector and the inner unit normal vector of M_t respectively, and $\mathcal{H}^{n-1}$ is the $(n-1)$-dimensional Hausdorff measure.

In 2016, Mugnai, Seis and Spadaro proved the existence of the global distributional solution for (0.3) by using a variational approach.

We consider the phase field method for (0.3). In 1992, Rubinstein and Sternberg studied (0.3) by using the Allen-Cahn equation with non-local term. However, whether the solution converges to the time global weak solution of (0.3) or not is an open problem, due to the difficulty of estimates of the Lagrange multipliers.

In this talk, we consider the Allen-Cahn equation with non-local term studied by Golovaty. The main results of this talk are the L^2-estimates of the Lagrange multipliers and the time global existence of the weak solution for (0.3) by using the Allen-Cahn equation with non-local term.

Traveling Waves in a PDE Model of Tumour Growth

Tohru Wakasa

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This talk is based on the joint work with Michiel Bertsch (Univ. Rome Tor Vertaga, Italy), Danielle Hilhorst (Univ. Paris Sud, France), Hirofumi Izuhara (Miyazaki Univ., Japan) and Masayasu Mimura (Meiji Univ., Japan).

A nonlinear PDE model for tumour growth, which takes account of contact inhibition, has been proposed by Bertsch, Dal-Passo and Mimura (2010). A feature of this PDE model is an appearance of traveling wave solution of a discontinuity, which is called as *segregated* (Bertsch-Mimura-Wakasa (2013)). On the other hand, this model also admits infinite number of smooth traveling wave solutions like the celebrated Fisher-KPP equation (Bertsch-Hilhorst-Izuhara-Mimura-Wakasa (2015)).

In this talk we will briefly introduce the PDE model, numerical results and analytical results on the traveling wave solutions, which are obtained in the articles above. Furthermore, recent works on the various kind of traveling wave solutions will be discussed.

On the Shape of Meissner Solutions to a Limiting Form of Ginzburg-Landau Systems

Xingfei Xiang

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In this talk we study a semilinear system involving the curl operator, which is a limiting form of the Ginzburg-Landau model for superconductors in $\mathbb{R}^3$ for a large value of the Ginzburg-Landau parameter. We consider the locations of the maximum points of the magnitude of solutions, which are associated with the nucleation of instability of the Meissner state for superconductors when the applied magnetic field is increased in the transition between the Meissner state and the vortex state. For small penetration depth, we prove that the location is not only determined by the tangential component of the applied magnetic field, but also by the normal curvatures of the boundary in some directions. We also show that the solutions decay exponentially in the normal direction away from the boundary if the penetration depth is small.

Uniqueness and Traveling Waves in a Cell Motility Model

Lei ZHANG

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We study the evolution equation for curves obtained as the sharp interface limit of a phase-field model for (crawling) motion of eukaryotic cells on a substrate. The sharp interface limit equation is non-linear and non-local. We establish uniqueness of solutions to the sharp interface limit equation in a so-called subcritical parameter regime. The proof relies on a Gronwall estimate for a specially chosen weighted L2 norm. This particular norm guarantees necessary control on non-local terms. Next, since persistent motion of crawling cells is of central interest to experimentalists we study the existence of traveling wave solutions. We prove that traveling wave solutions exist in the supercritical parameter regime provided the non-linear term of the sharp interface limit equation possesses certain asymmetry (related, e.g., to myosin contractility). Finally, we numerically investigate traveling wave solutions and simulate their dynamics. There is inherit difficulty due to non-uniqueness of solutions of the sharp interface limit equation. To resolve this issue we simulate a regularized, uniquely solvable PDE system. Our simulations suggest instability of traveling wave solutions and predict both bipedal wandering cell motion as well as rotating cell motion; these results qualitatively agree with recent experimental and theoretical findings.

Short Communications

Spatial Heterogeneity and Time Periodicity in Lotka-Volterra Competition-Diffusion Systems

Xiaoqing He

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In this talk I shall report some of the recent progress on the 2×2 Lotka-Volterra competition-diffusion systems when spatial heterogeneity and/or temporal periodicity are present. This is a joint work with Dr. Xueli Bai and Prof. Wei-Ming Ni.

Conformal Metrics in $\mathbb{R}^{2m}$ with Constant Q-Curvature and Arbitrary Volume

Xia Huang

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In this talk, we will report the polyharmonic problem $\Delta^m u = \pm e^u$ in $\mathbb{R}^{2m}$, with $m \geq 2$. In particular, we prove that *for any* $V > 0$, there exist radial solutions of $\Delta^m u = -e^u$ such that

$$\int_{\mathbb{R}^{2m}} e^u dx = V.$$

It implies that for m odd, given any $Q_0 > 0$ and arbitrary volume $V > 0$, there exist conformal metrics g on $\mathbb{R}^{2m}$ with constant Q-curvature equal to Q_0 and $\mathrm{vol}(g) = V$. This answers some open questions in Martinazzi's work. This is a joint work with Prof. Dong YE.

Qualitative Analysis on a Diffusive SIS Epidemic Model with Varying Total Population

Huicong Li

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We consider an SIS epidemic reaction-diffusion system with a linear source in spatially heterogeneous environment. The main feature of our model lies in that its total population varies, compared to its counterpart proposed by Allen et al (Asymptotic profiles of the steady states for an SIS epidemic reaction-diffusion model, Discrete Contin. Dyn. Syst., 21 (2008), 1–20). The uniform bounds of solutions are derived, based on which, the threshold dynamics in terms of the basic reproduction number is established and the global stability of the unique endemic equilibrium is discussed when spatial environment is homogeneous. In particular, the asymptotic profile of endemic equilibria is determined if the diffusion rate of the susceptible or infected population is small or large. The theoretical results show that a varying total population can enhance persistence of infectious disease, and therefore the disease becomes more threatening and harder to control. This is a joint work with Rui Peng and Feng-Bin Wang.

Steady States of Hall-MHD System

Yong Zeng

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In this talk, we focus on the steady states of the resistive incompressible Hall magneto-hydrodynamics (Hall-MHD) system, with emphasis on the effect of the Hall term on the

solutions. More precisely, we study the following steady Hall-MHD system

$$\begin{cases} \operatorname{curl}(\operatorname{curl}\mathbf{B}+\mu\operatorname{curl}\mathbf{B}\times\mathbf{B}-\mathbf{u}\times\mathbf{B}+\mathbf{f})=\mathbf{0} & \text{in } \Omega,\\ -\Delta\mathbf{u}+(\mathbf{u}\cdot\nabla)\mathbf{u}+\nabla p-\operatorname{curl}\mathbf{B}\times\mathbf{B}=\mathbf{g} & \text{in } \Omega,\\ \operatorname{div}\mathbf{B}=\operatorname{div}\mathbf{u}=0 & \text{in } \Omega,\\ \mathbf{B}_T=\mathbf{0},\quad \mathbf{u}=\mathbf{0} & \text{on } \partial\Omega. \end{cases} \tag{$*$}$$

Existence of H^1 weak solutions to the above problem $(*)$ is proved for a general domain. We show that it has infinitely many H^1 weak solutions if Ω has holes, which are parameterized by elements in $\mathbb{H}_2(\Omega)$. Existence of H^2 solutions to $(*)$ is proved for either small data $(\mathbf{f},\mathbf{g})\in H^1(\Omega,\mathbb{R}^3)\times L^2(\Omega,\mathbb{R}^3)$ or small Hall parameter μ. The asymptotic behavior of the solutions in the vanishing Hall parameter limit ($\mu\to 0$) is examined, and the asymptotic stability of the steady states is also investigated.

Existence and Regularity of Weak Solutions to a Thermoelectrical Model

Zhibing Zhang

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This paper is devoted to study existence, regularity and uniqueness of weak solutions of the following system

$$\begin{cases} \operatorname{curl}[\rho(u)\operatorname{curl}\mathbf{H}]=\mathbf{0} & \text{in } \Omega,\\ -\Delta u=\rho(u)|\operatorname{curl}\mathbf{H}|^2 & \text{in } \Omega,\\ \operatorname{div}\mathbf{H}=0 & \text{in } \Omega,\\ u=u^0,\quad \mathbf{H}_T=\mathbf{H}^0_T & \text{on } \partial\Omega. \end{cases}$$

Here Ω is a bounded domain in $\mathbb{R}^3$ with a C^2 boundary $\partial\Omega$, u is a scalar function and $\mathbf{H}$ is a vector field. The notation $\mathbf{H}_T=\mathbf{H}-(\nu\cdot\mathbf{H})\nu$ denotes the tangential component of $\mathbf{H}$, where ν is the unit outer normal vector on $\partial\Omega$. $\mathbf{H}$ represents the magnetic field and u represents the temperature, $\rho(u)$ represents the electrical resistivity of the material. Existence and regularity of weak solutions are obtained for general boundary data, while uniqueness is established for small boundary data. Several related systems are also studied. This is a joint work with Prof. Xingbin Pan.