International Conference on Complex Geometry and Several Complex Variables

May 11 to May 15, 2015

ABSTRACTS

Speaker: **Dan Burns**
Title: *Gauss-Bonnet for currents directed by a singular foliation by Riemann surfaces and applications*
Abstract: TBA

Speaker: **Bo-Yong Chen**
Title: *The Bergman metric and isoperimetric inequalities*
Abstract: It is well-known that every Riemannian surface carries at least one complex structure. Thus it reasonable to do quantitative complex analysis on a Riemannian surface through the underlying geometry of the Riemannian metric. The purpose of this talk is to study the Bergman metric on Riemann surfaces by using isoperimetric inequalities.

Speaker: **Dan Coman**
Title: *Equidistribution for sequences of line bundles on normal Kähler spaces*
Abstract: We study the asymptotics of Fubini-Study currents and zeros of random holomorphic sections associated to a sequence of singular Hermitian line bundles on a compact normal Kähler complex space. This is a generalization of our previous results in two directions: we allow the base space to be singular and we consider sequences (L_p, h_p) of singular Hermitian holomorphic line bundles whose Chern curvature satisfy a natural growth condition, instead of sequences of powers of a fixed line bundle (L,h). The results are joint with X. Ma and G. Marinescu.

Speaker: **Jean-Pierre Demailly**
Title: *On the Green-Griffiths-Lang and Kobayashi conjectures for the hyperbolicity of algebraic varieties*
Abstract: The Green-Griffiths-Lang conjecture stipulates that for every projective variety X of general type over $\mathbb{C}$, there exists a proper algebraic

subvariety of X containing all non constant entire curves $f : \mathbb{C} \to X$. Using the formalism of directed varieties, we prove that this assertion holds true in case X satisfies a strong general type condition that is related to a certain jet-semistability property of the tangent bundle T_X. We then explain how this result can be used to investigate the long-standing conjecture of Kobayashi (1970), according to which every general algebraic hypersurface of dimension n and degree at least $2n + 1$ in the complex projective space $\mathbb{P}^{n+1}$ is hyperbolic.

Speaker: **Tien-Cuong Dinh**
Title: *Equidistribution speed for Fekete points associated with an ample line bundle*
Abstract: Let K be a compact subset with smooth boundary in C^n. A Fekete configuration of order p for K is a finite subset of K maximalizing the Vandermonde determinant associated with polynomials of degree p or less than p. A recent theorem by Berman, Boucksom and Witt Nystry implies that Fekete configurations for K are asymptotically equidistributed with respect to a canonical equilibrium measure for K when p tends to infinity. We give an explicit estimate for the speed of convergence. The result also holds in a general setting of Fekete points associated with an ample line bundle over a projective manifold. This is a join work with Xiaonan Ma and Viet-Anh Nguyen.

Speaker: **Peter Ebenfelt**
Title: *Sums of squares and applications to projective and CR geometry*
Abstract: We will discuss a particular sums of squares problem for analytic functions, and explore how progress on this problem yields results concerning mappings between projective spaces and mappings between spheres in complex spaces.

Speaker: **Lawrence Ein**
Title: *Syzygies of algebraic varieties*
Abstract: We'll discuss various conjectures relating geometry of algebraic varieties to syzygies of these varieties.

Speaker: **Siqi Fu**
Title: *Spectral properties of the $\bar{\partial}$-Neumann Laplacian*
Abstract: The $\bar{\partial}$-Neumann Laplacian is an elliptic operator with non-coercive boundary conditions. Its spectrum is more sensitive to underlying geometric

and analytic structures than the usual Dirichlet or Neumann Laplacian. In this talk, we discuss interplays between spectral behavior of the $\bar{\partial}$-Neumann Laplacian and geometric and analytic properties of the underlying manifold. In particular, we study stability of the spectrum as the underlying geometric and analytic structures deform.

Speaker: **Xianghong Gong**
Title: *The $\bar{\partial}$-equation on variable strictly pseudoconvex domains*
Abstract: We investigate regularity of a family of solutions of the $\bar{\partial}$-equation on a family of pseudoconvex domains that depend on a parameter. We obtain the interior regularity in terms of the parameter for solutions on pseudoconvex domains. We also obtain the boundary regularity in terms of the parameter for solutions on strongly pseudoconvex domains. This is joint work with Kang-Tae Kim.

Speaker: **vincent Guedj**
Title: *Weak Kähler-Ricci flows*
Abstract: Studying the long-term behavior of the Kähler-Ricci flow on mildly singular varieties, one isnaturally lead to construct weak solutions of degenerate parabolic complex Monge-Ampère equations. In this talk we develop a viscosity theory for such degenerate flows which allows in particular to define and study the (normalized) Kähler-Ricci flow on varieties with canonical singularities. This is joint work with P. Eyssidieux and A. Zeriahi.

Speaker: **ILYA KOSSOVSKIY**
Title: *New extension phenomena for automorphisms of real-analytic hypersurfaces*
Abstract: The problem of regularity of mappings between real hypersurfaces in complex space has attracted a lot of attention of experts in CR-geometry in the last few decades. Working in this direction, we discovered in our 2014 work with Lamel the surprising phenomenon of existence of real-analytic hypersurfaces in complex space which are C^∞ but no analytically CR-equivalent (thus giving a counterexample to an analyticity property conjectured previously by Ebenfelt and Huang). This work revived the interest to the study of the nature of CR-mappings, and raised again the general question:"What CR-maps actually are"? In our new work with Lamel, we provide a solution for this general problem in the case of (infinitesimal generators of) automorphisms of real-analytic hypersurfaces in complex two-space.

First, we discover the "sectorial analyticity" phenomenon for smooth CR-automorphisms. That is, we prove the analytic extension of infinitesimal automorphisms to certain sectorial domains associated with a hypersurface. Importantly, the extended automorphisms admits an asymptotic power series representation in these sectorial domains. Second, we find an (explicit!) optimal condition for hypersurfaces guaranteeing that their infinitesimal automorphisms extend analytically to a neighborhood of the reference point. We call this condition "Fuchsian type". Both results are proved using the new Dynamical technique (the CR - DS technique) introduced in our joint work with Shafikov and Lamel)

Speaker: **Uros Kuzman**
Title: *Gluing techniques and envelopes of disc functional on almost complex manifolds*
Abstract: We will discuss pseudoholomorphic maps on manifolds equipped with a non-integrable almost complex structure. In particular, we present deformation theory for J-holomorphic disc based on the singular integral theory, some approximation results and Donaldson's gluing method for such maps.Moreover, we give an application of these techniques into the theory of disc functionals and prove J-plurisubharmonicity of the envelope of Poisson functional.

Speaker: **Song-Ying Li**
Title: *Super-Rigidity theorem on pseudo-Hermitian CR manifolds*
Abstract: This is a preliminary report of a joint work with Duong Ngoc Son. We are working on CR-analogous of the Siu's super rigidity theorem for harmonic map from a Riemannian manifold to a strongly negative Kähler manifold.

Speaker: **Ngaiming Mok**
Title: *From transcendence to algebraicity: techniques of analytic continuation on bounded symmetric domains and their dual compact Hermitian symmetric spaces*
Abstract: Analytic continuation is a central issue in Several Complex Variables, starting with the Hartogs Phenomenon. We examine the applications of techniques of analytic continuation in Complex Geometry for irreducible bounded symmetric domains Ω and their dual Hermitian symmetric spaces of the compact type S, and their ramifications to the geometric theory of uniruled projective manifolds. As a starting point, in the case where $\text{rank}(S) \geq 2$

we recall a proof of Ochiai's theorem (1970) for analytic continuation of flat S-structure using Hartogs extension, and its generalization to the Cartan-Fubini extension principle of Hwang-Mok (2001) in the geometric theory of uniruled projective manifolds basing on varieties of minimal rational tangents (VMRTs). Applying methods of algebraic extension in CR-geometry of Webster and Huang, and Ochiai's theorem, we give the proof of Mok-Ng (2012) that under a nondegeneracy assumption, a germ of measure-preserving holomorphic map $f : (\Omega, \lambda d\mu_\Omega; 0) \to (\Omega, d\mu_\Omega; 0) \times \cdots \times (\Omega, d\mu_\Omega; 0)$, where $d\mu_\Omega$ denotes the Bergman volume form and $\lambda > 0$ is a real constant, is necessarily a totally geodesic diagonal embedding, answering in the affirmative a problem of Clozel-Ullmo stemming from a problem in Arithmetic Dynamics regarding Hecke correspondences. The proof involves Alexander's Theorem for the complex unit ball B^n, $n \geq 2$, in the rank-1 case and a new Alexander-type extension theorem for the case of irreducible bounded symmetric domains Ω of rank ≥ 2 for germs of holomorphic maps preserving the regular part $\mathrm{Reg}(\partial\Omega)$ of the boundary. In another direction we explain the non-equidimensional Cartan-Fubini extension principle of Hong-Mok (2010) and its application to the characterization of smooth Schubert varieties in rational homogeneous manifolds of Picard number 1 (Hong-Mok 2013). Finally, we consider the problem of analytic continuation of subvarieties of uniruled projective manifolds $(X, \mathcal{K})$ equipped with a VMRT-structure (e.g. irreducible Hermitian symmetric spaces S of the compact type) under the assumption that the subvariety inherits a sub-VMRT structure by taking intersections of VMRTs with tangent spaces, and establish a principle of analytic continuation (Mok-Zhang 2015) by a parametrized Thullen extension of sub-VMRT structures along chains of rational curves.

Speaker: **Sui-Chung Ng**
Title: *Geometry of various mapping problems of symmetric domains*
Abstract: We will look at various mapping problems of symmetric domains, including those between bounded symmetric domains and also between certain unbounded symmetric domains on Grassmannians. While some problems are by definition geometric in nature, namely, holomorphic isometries and measure-preserving mappings (with respect to the Bergman metric), we will also present a geometric approach to deal with proper holomorphic mappings. For problems related to the Bergman metric, we will explain their original motivations and see how they are linked to Arithmetic Geometry. And for proper holomorphic maps, we are going to demonstrate how the

moduli structures of certain special subspaces of the symmetric domains can be exploited to force rigidity in the mappings.

Speaker: **Junjiro Noguchi**
Title: *On Oka's coherence theorems, Oka VII, VIII, some improvements and related results.*
Abstract: This is a survey talk of Oka's work on his coherence theorems with some improvements and new results. Oka proved three coherence theorems in Oka VII and VIII Here there are two versions of the paper Oka VII, one in Bull. Soc. Math. France (1950) and the other in his collected works, Iwanami, Tokyo (1961). It is known that the first one was modified by H. Cartan, which was different from the original second one published from Iwanami. We discuss the differences among them and what Oka was willing to do. It was related to the Levi Problem (Hartogs' Inverse Problem): In fact, he wrote a research report in Japanese to solve the problem for Riemann domains over C^n in 1943, just after Oka VI (1942) solving it in the case of two dimensional univalent domains. We will discuss the coherence of ideal sheaves of analytic subsets by Oka VIII and H. Cartan (1950), give some improvements of the proofs of Oka's coherence theorem,Oka's Theorem (Oka IX), Behnke-Stein's Theorem, and apply them to ramified Riemann domains over C^n.

Speaker: **Yakeo Ohsawa**
Title: *Variation of complex structure and L^2 extension theorem*
Abstract: Since Riemann, the theory of variations of complex structures has developed, through the works of Teichmuller and Kodaira-Spencer, into a modern field involving a forefront like conformal field theory. Griffiths studied the periods of Riemann surfaces in a generalized setting of the variation of Hodge structures, in connection to the higher dimensional analogue of Torelli's theorem. The point of his theory is described as a transversality theorem for a canonically defined connection. By exploiting this, T. Fujita has established a semipositivity theorem for the direct image of relative canonical bundles. Last year, in a work by Berndtsson and Lempert, a remarkable connection was found between this kind of semipositivity and the so-called Ohsawa-Takegoshi L^2 extension theorem. A story around this development will be reviewed. A new straightforward proof of the sharp L^2 extension theorem will also be presented.

Speaker: **Jean Ruppenthal**

Title: *L^2-cohomology of canonical singularities and integral representation formulas*

Abstract: Many L^2-$\bar\partial$-cohomology groups of singular complex spaces can be described in terms of a resolution of singularities. This way was first pursued successfully by Pardon and Stern [PS], but considerable progress has been made also just recently by vrelid and Vassiliadou [OV] and in [R1], where resolutions of singularities are used to give smooth models for the L^2-cohomology with respect to various $\bar\partial$-operators. By use of L^2–Serre duality for $\bar\partial$-cohomology classes, [R2], one obtains an even more complete picture.

In general, there exist local obstructions to solvability of $\bar\partial$-equations in singular points. Here singularities as appearing in the minimal model program come into play. It is shown in [R2] that the $\bar\partial$-equation is solvable in some L^2 -sense for $(0,q)$-forms and (n,q)-forms at canonical singularities (let n be the dimension of the complex space). However, many questions remain open, and it seems that completely new techniques are needed besides L^2-theory for the $\bar\partial$-operator.

One hope lies in the integral representation formulas for singular complex spaces developed recently by Andersson and Samuelsson [AS] who introduced Koppelman formulas (i.e., $\bar\partial$-homotopy formulas) for singular complex spaces. Clearly, these $\bar\partial$-homotopy formulas cannot hold in the L^2-sense in general because we know that there exist obstructions to solving the $\bar\partial$-equation in the L^2 -sense. Actually, the integral operators of Andersson-Samuelsson are defined only as principal value integrals which must be applied to smooth forms (and something derived from smooth forms, the so-called $\mathcal{A}$-sheaves). They cannot be applied to L^2-forms on arbitrary varieties.

But, if the $\bar\partial$-equation is solvable in the L^2-sense (as for canonical singularities), then there is a good hope that the solution can be provided explicitly by the Andersson-Samuelsson operators. Actually, it turns out that canonical singularities have a "good" influence on the singularity of the integral kernels in the Andersson Samuelsson operators. Besides the BMK-part, $\|xi-z\|^{1-2n}$, the singularity of the integral kernels consists of the so-called structure form which measures the "badness" of the singularity. These structure forms are less harmful for canonical singularities. So, it appears reasonable to study the mapping properties of the Andersson-Samuelsson operators for canonical singularities.

We will report on some results in that direction. This is a joint project with Mats Andersson, Richard Lärkäng, Hakan Samuelsson-Kalm and Elizabeth Wulcan. At the moment, we can actually give L^p-estimates (for

$2 \leq p \leq \infty$)) and $\bar{\partial}$-homotopy formulas for the Andersson-Samuelsson operators for canonical surface singularities (so-called du Val-singularities) and rational cones. For the A_1-singularity, this can be found in [LR].

References.

AS M. Andersson and H. Samuelsson, A Dolbeault-Grothendieck lemma on complex spaces via Koppelman formulas, *Invent. Math.* **190** (2012) 261–297.

LR R. Lärkäng and J. Ruppenthal, Koppelman formulas on the A_1-singularity, Preprint 2014, arXiv:1407.5703, submitted.

OV N.Øvrelid and S. Vassiliadou, L^2-$\bar{\partial}$-cohomology groups of some singular complex spaces, *Invent. Math.* **192** (2013), no.2, 413–458.

PS W. Pardon and M. Stern, L^2-$\bar{\partial}$-cohomology of complex projective varieties, *J. Amer. Math. Soc.* **4** (1991), no. 3, 603–621.

R1 J. Ruppenthal, L^2-theory for the $\bar{\partial}$-operator on compact complex spaces, *Duke Math. J.* **163** (2014), 2887–2934.

R2 J. Ruppenthal, L^2-Serre duality on singular complex spaces and rational singularities, Preprint 2014, arXiv:1401.4563, submitted.

Speaker: **Giuseppe Della Sala**

Title: *Analytic transformations of smooth submanifolds*

Abstract: We are interested in the following general question: how does the (pseudo)group of local, complex or real-analytic diffeomorphisms of the Euclidean space act on submanifolds which are smooth, but not necessarily real-analytic? More in particular, what can be said about the orbits of such actions? It turns out that various interesting phenomena can arise. We will discuss some of them, presenting some recent results and open problems.

Speaker: **Mei-Chi Shaw**

Title: *Topology of Dolbeault cohomology group*

Abstract: The range of the Cauchy-Riemann operator for a domain in a complex manifold X is well understood if the complex manifold X is a Stein manifold. Much less is known when the domain or the manifold is not Stein. In this talk we will discuss some recent results on the Hausdorff property of Dolbeault cohomologies. In particular, we give an example of a pseudoconvex

Stein domain Ω with smooth boundary in a compact complex Hermitian manifold X whose range of $\bar{\partial}$ is not closed in L^2. These results are joint work with Debraj Chakrabarti and Christine Laurent-Thiebaut.

Speaker: **Bernard Shiffman**
Title: *L^p norms of holomorphic sections on compact Kähler manifolds*
Abstract: We consider sections s_k in the spaces $SH^0(M, L^k)$ of L^2-normalized sections of the tensor powers of a positive holomorphic line bundle L on a compact Kähler manifold M. In joint work with Zelditch in 2003, we showed that for $2 < p < \infty$, the average L^p norms of the s_k are bounded independently of k, and that the probability of deviation from the median decays rapidly. However, the average sup-norms are unbounded and grow at the order $\sqrt{\log k}$. We shall show how to construct sequences of orthonormal sets of sections of $SH^0(M, L^k)$ with cardinality of the order k^m which are uniformly bounded independently of k. These results make use of the asymptotics of the Bergman kernel. An improvement on the cardinality was recently given by Ortega-Cerda, and Bourgain has constructed uniformly bounded orthonormal bases for the cases $M = CP^1$ and CP^2.

Speaker: **Yum-Tong Siu**
Title: *Splitting of unstable plane bundle over 6-dimensional projective space*
Abstract: Will present a proof of the splitting of unstable holomorphic vector bundles of rank 2 over the complex projective space of dimension 6. The new techniques used in the proof require the stronger assumption of dimension 6 for the complex projective space than the conjectured dimension of 4 or 5.

Speaker: **Sai Kee Yeung**
Title: *Geometry of a complex two ball quotient, Cartwright-Steger surface*
Abstract: The purpose of the talk is to explain a joint work with Vincent Koziarz and Donald Cartwright on the geometry of a complex two ball quotient constructed earlier by Cartwright and Steger. The surface is the unique complex surface of Euler number 3 which is neither the projective plane nor a fake projective plane. Geometric properties of the surface such as the genus of the Albanese mapping are difficult to understand from the conventional methods. We would address some problems in this direction and use it to understand other open problems related to complex ball quotients.

Speaker: **Xianyu Zhou**
Title: *TBA*

Abstract: TBA