《ECNU 2014 Hopf 代数、量子群与表示论暑期学校及工作营》

ECNU Summer School and Workshop on Hopf Algebras, Quantum Groups and Representation Theory, 2014

Organized by the Advanced Research Centre of Algebra

　　2014.7.28---8.8, 华东师范大学数学系拟将举办研究主题为《Hopf 代数、量子群与表示论》的为期两周的研究生暑期学校，含 2 天的学术会议。Kaplansky 1975 年在 Hopf 代数领域提出的 10 个猜想一直引领着该领域的发展，近 10 多年来，随着量子群发展步入新阶段，也推动着 Hopf 代数领域尤其是分类工作的迅猛发展，其研究方法、思想手段、观点看法的更新和深入日新月异，近年来最重大的突破是 Andruskiewitsch-Schneider 完成了"以有限 Abel 群代数为余根基的有限维点 Hopf 代数分类工作"---2010 年发表在国际顶级数学刊物美国数学年刊 Annals Math.上，引发了新的国际研究热点。

　　华东师大数学系胡乃红教授及代数研究中心为本次暑期学校邀请了工作在 Hopf 代数、量子群和表示论国际前沿的著名专家和学者：Hopf 代数分类学权威专家来自阿根廷的 Andruskiewitsch 教授、量子群著名专家巴黎七大 Rosso 教授、澳大利亚悉尼大学著名数学物理及不变量专家张瑞斌教授、俄罗斯著名模李代数及 Hopf 代数专家 Skryabin 研究员，分别来讲授每人 8 小时的短课程，共计 32 学时。暑期班将开设以下课程：

Mini-Courses:
(1) On Classification of Pointed Hopf algebras, by N. Andruskiewitsch;
(2) Cofree Hopf algebras and quantum groups, by M. Rosso;
(3) Hopf algebras and their Actions, by S. Skryabin;
(4) Introduction to Lie superalgebras and their representations, by R.B. Zhang

Outlines of mini-courses [课程内容介绍]:

(1) On Classification of Pointed Hopf Algebras

1. Nichols algebras. The braid equation. Braided vector spaces and Yetter-Drinfeld modules. Alternative definitions of Nichols algebras. Basic
examples. Approximations of Nichols algebras.
2. Nichols algebra of diagonal type. The PBW theorem of Kharchenko. The classification of Heckenberger. Relations with Lie (super) algebras.
3. The Weyl groupoid. Coxeter groupoids. Crystallographic data and Weyl

groupoid data. Outline of the classification. Convex orders.

4. Defining relations for Nichols algebra of diagonal type. Convex orders. Quantum Serre relations and their generalizations; powers of root vectors relations.

5. The lifting method. Hopf algebras generated by the coradical. The coradical filtration and the standard filtration. The associated graded Hopf algebras. Bosonization and the role of Nichols algebras.

6. Deformations of Nichols algebras. The general strategy. Classification results for pointed Hopf algebras with abelian group.

7. Nichols algebra of group type. Racks and cocycles. Classification of simple racks. Examples of finite-dimensional Nichols algebras of group type. The Fomin-Kirillov algebras.

8. Pointed Hopf algebras with non-abelian group. The collapsing criteria. Classification results for pointed Hopf algebras with non-abelian group.

(2) Cofree Hopf Algebras and Quantum Groups

Abstract: Connected cofree Hopf algebras have a universal property allowing the construction of many compatible Hopf algebra structures. They were classified by J-L Loday and M. Ronco, and familiar examples include shuffle Hopf algebras and quasi-shuffle Hopf algebras which appear in many domains of mathematics: combinatorics, number theory (multiple zeta values), Rota-Baxter algebras, ...
Replacing the ground field by a Hopf algebra H leads to a wide extension of the framework; the relevant category is that of Hopf bimodules M over H, and for each M, one can associate a natural (not connected) cofree coalgebra, first introduced by W. Nichols. The classification of compatible Hopf algebra structures leads, in particular examples, to quantum quasi shuffle algebras and to a new construction of quantized envelopping algebras. This provides a new framework to construct representations.

(3) Hopf Algebras and their Actions

Abstract.   Hopf algebras have found important applications in various areas   of mathematics. At the same time the structural properties of Hopf Algebras remain far from being fully understood.This series of lectures will start at the basics of the theory and will ove gradually on towards deeper results describing ring-theoretic properties of Hopf algebras, their actions and coactions on associative algebras. Particular questions discussed are conditions ensuring that a Hopf module algebra is Frobenius or quasi-Frobenius, existence of classical quotient rings for Hopf module algebras, extension of the module structure to quotient rings, projectivity and faithful flatness of Hopf algebras over Hopf subalgebras and right coideal subalgebras. An important tool in the study are equivariant and coequivariant modules.

(4) Introduction to Lie Superalgebras and their Representations

Outline：

1. Lie superalgebras

The general linear Lie superalgebra gl(m|n), orthosymplectic Lie superalgebra osp(m|2n); simple Lie superalgebras of classical type.

2. Invariant theory

Tensor representations of gl(m|n), first fundamental theorem of invariant theory for gl(m|n), Schur-Weyl duality, a super duality; tensor representations of osp(m|2n), Schur-Weyl-Brauer duality.

3. Parabolic category **O** of gl(m|n)

Parabolic category **O;** canonical bases of quantum gl(∞); Kazhan-Lusztig polynomials of gl(m|n); a closed character formula and dimension formula; Jantzen filtration for Kac modules.

4. Finite dimensional representations of osp(m|2n)

Flag supermanifolds; elements of Bott-Borel-Weil theory for osp(m|2n); a combinatorial algorithm for computing characters.

Lectures Schedule (Teaching Building 3, Room 202)

	July 28	July 29	July 30	July 31	Aug. 1
9:00---11:15 (with 15' break)	Skryabin	Rosso	Andrusk.	Rosso	Skryabin
14:45---17:00 (with 15' break)	Rosso	Skryabin	Rosso	Andrusk.	Andrusk.

	Aug. 4	Aug. 5	Aug. 6	Aug. 7	Aug. 8
9:00---11:15 (with 15' break)	Zhang	Andrusk.	Zhang	Conference	
14:45---17:00 (with 15' break)	Skryabin	Zhang	Zhang		

Conference Schedule (Teachers' Home Hotel, 3rd Flour)

	August 7	August 8
8:30---9:20	Andruskiewitsch	R. B. Zhang
9:30---10:00	Jiao Zhang	Xun Xie
10:00---10:30	Tea Time	

10:30---11:00	Yunnan Li	Ming Liu
11:30---12:30	Lunch Time	
14:00---14:50	Skryabin	Quanshui Wu
15:00---15:30	Honglian Zhang	Xiaoyu Chen
15:30---16:00	Tea Time	Tea Time
16:00---16:50	Bin Shu	Guodong Zhou

Program of Conference

Titles and Abstracts:

Generalized root systems, contragredient Lie (super)algebras and Nichols algebras of diagonal type
by N. Andruskiewitsch   (Ciudad University, Argentina)
Abstract: I will review the definition of *Generalized root systems* (due to Heckenberger and Yamane). I will show how to attach a Generalized root systems to a *Contragredient Lie (super)algebra* and to a *Nichols algebras of diagonal type,* and discuss the relations between these to assignements.

The BLM realization for the integral form of quantum supergroup $U_v(gl_{m|n})$

by Xiaoyu Chen   (ECNU and Institute of Mathematics, Chinese Academy of Sciences)
Abstract: Let U be the Lusztig integral form of the quantized enveloping superalgebra $U_v(gl_{m|n})$ of general linear Lie superalgebra $gl(m|n)$,   and let $S(m|n,r)$ be the integral quantum Schur superalgebra. Let $S(m|n) = \prod_{r \geq 0} S(m|n,r)$. In this talk, we give a "super analogue" of the BLM realization, i.e., we realize U as a certain $Z[v,v^{-1}]$ - subalgebra of $S(m|n)$. As an application, we give the BLM realization for quantum supergroup $U_v(gl_{m|n})$ with v an odd root of unity.

Representation rings and Hopf 2-cocycle twists --- the 2-rank Taft algebra
by Yunnan Li   (South China University of Technology)
Abstract: Recently, many works for pointed Hopf algebras over non-abelian groups demonstrate that the lifting method to construct new pointed Hopf algebras are essentially based on Hopf 2-cocycle twists. In this talk, we want to emphasize that representation rings of cocycle deformations of Hopf algebras can vary dramatically, in contrast with the monoidal equivalence of their comodule categories. As the main example, we introduce detail information about representation rings of the 2-rank Taft algebra and its relative cocycle twists. This is a joint work with Naihong Hu.

R-matrix realization of two-parameter quantum group $U_{r;s}(\mathrm{gl}_n)$
by Ming Liu   (Nankai University and South China University of Technology)

Abstract: We provide a Faddeev-Reshetikhin-Takhtajan's RTT approach to the quantum group $Fun(GL_{r;s}(n))$ and the quantum enveloping algebra $U_{r;s}(\mathrm{gl}_n)$ corresponding to the two-parameter R-matrix. For $Fun(GL_{r;s}(n))$, we prove that the quantum determinant $det_{r;s}T$ is a quasi-central element in $Fun(GL_{r;s}(n))$. For $U_{r;s}(R)$, we construct $n - 1$ quasi-central elements in $U_{r;s}(R)$ which are analogues of higher Casimir elements in $U_q(\mathrm{gl}_n)$. We prove that the $U_{r;s}(\mathrm{gl}_n)$ can be derived from the R-matrix realization $U_{r;s}(R)$ by using Ding-Frenkel's method.

Noetherian Hopf module algebras
by S. Skryabin   (Kazan Federal University, Russia)

Abstract: In this talk I am going to discuss some of my recent results on ring-theoretic properties of Noetherian associative algebras with a given action of a Hopf algebra.

The centers of the universal enveloping algebras of classical Lie superalgebras in prime characteristic
by Bin Shu   (East China Normal University)

Abstract: For a classical Lie superalgebra over an algebraically closed field of characteristic bigger than 2, which is a Lie superalgebra of a supergroup with the purely even subgroup being a reductive group.   In this talk, we show that the center of the universal enveloping algebra is an integral domain, and the fraction algebra of the enveloping algebra is a simple algebra over the fraction field of the integral domain. We then prove that the fraction field of the center coincides with that of the subalgebra generated by the invariant ring of the center under the purely even group and the Frobenius center. In the case osp(1|2n), the center is more precisely described. Furthermore, we will demonstrate the precise relation between the smooth points of the maximal spectrum of the center and the corresponding irreducible modules for osp(1|2). This is a joint work with Junyan Wei and Lisun Zheng.

Hopf action on Calabi-Yau algebras
by Quanshui Wu   (Fudan University)

Abstract: In this talk, we will prove the following main theorem: Let H be an involutory Calabi-Yau Hopf algebra and A be a left-graded H-module algebra. If A is a p-Koszul ($p \geq 2$) Calabi-Yau algebra, then $A\#H$ is a graded Calabi-Yau algebra if and only if the homological determinant of the Hopf action on A is trivial. To this end, we prove a Van den Bergh Duality Theorem for Hopf actions which yields an isomorphism between the Hochschild cohomology groups $HH_i(A,N)$ and the Hochschild homology groups $HH_{d-i}(A,\mathrm{Ext}_{iA_e}(A,A_e)\otimes_A N)$ for an involutory Hopf algebra H, a left-graded H-module algebra A satisfying some homological smoothness conditions and any $A\#H$-bimodule N. This is a joint ork with Liyu Liu and Can Zhu.

Graded homogeneous right coideal subalgebras for generalized quantum algebras
by Xun Xie (Institute of Mathematics, Chinese Academy of Sciences)

Abstract: We will describe the set of graded homogeneous right coideal subalgebras for generalized quantum algebras (=Drinfeld double of the bosonization of the Nichols algebras of diagonal type) of finite type, which generalizes the result of Heckenberger on graded homogeneous right coideal subalgebras for classical quantum groups.

Hopf algebraic structures of quantum algebras
by Honglian Zhang (Shanghai University)

Abstract: In this talk, we consider to use a new set of simpler generators of quantum affine algebras to replace the original generators, and this also enables us to give a Hopf algebraic structure for the Drinfeld realization of the quantum affine algebras. Furthermore, we study the Hopf algebraic structure of quantum toroidal algebra for type A. The method works well for both one-parameter case and two-parameter case.

On cyclic homology of strong smash product algebras
by Jiao Zhang (Shanghai University)

Abstract: In this talk, we study the cyclic homology of strong smash product algebras, which contain many kinds of products of two algebras. A special sequence is established to converge to the cyclic homology. Examples are provided to show how our results work.

Fundamental theorems of invariant theory for classical supergroups
by R.B. Zhang (University of Sydney, Australia)

Abstract: The first and second fundamental theorems (FFT and SFT) of invariant theory for the general linear supergroup $GL_{m|l}$ and orthosymplectic supergroup $OSp_{m|2n}$ are establisbed. The theorems reduce to FFTs and SFTs of invariant theory for GL_m in the special case of $GL_{m|l}$ with l=0, and for O_m and Sp_{2n} in the special cases of $OSp_{m|2n}$ with n=0 or m=0 respectively. The proof of the SFT for $OSp_{m|2n}$ yields new proofs of the SFTs for O_m and Sp_{2n}. This is joint work with Gus Lehrer.

On the Hochschild cohomology ring of a finite dimensional Hopf algebra
by Guodong Zhou (East China Normal University)

Abstract: We show that the Hochschild cohomolgy ring of a Frobenius algebra with semisimple Nakayama automorphism is a Batalin-Vilkovisky algebra, so the Hochschild cohomology ring of a finite dimensional Hopf algebra defined over an algebraically closed field of characteristic zero is a BV algebra. We also explain how the proof of the above result also indicates a reduction technique for a folklore conjecture about Hopf algebras: the ususal cohomology ring of a finite dimensional Hopf algebra is finitely generated.

[程序委员会] 胡乃红 (Coordinator)、时俭益、舒斌、谈胜利、王建磐

[暑期学校规模] 本暑期学校预设的听众是本校代数方向的感兴趣的硕士生、博士生、博士后和青年教师，以及本校毕业的相关方向的青年教师和博士后，并接受部分兄弟院校相关方向的博士研究生，总的听众规模约 50 人，已接纳外校报名博士生 25 人，住教师招待所、研究生公寓；由于本次活动经费所限，外校青年教师及博士后人员的住宿费自理。

目前已不再接受额外的与会注册。

[致外校学员的注意事项]

1. 临时校园卡我们已经在网上提交申请了。学员来校后，请本人带好身份证和 20 元工本费前往校图书馆校园卡中心拍照领卡。同时，请注意：卡不要丢弃，如果以后再参加数学系活动的话，该卡由我系帮忙激活后可以再次使用。

2. 安排住在教师招待所的老师，请自行前往研究生公寓 12 楼（闵行区红梅南路 5800 号），凭身份证登记入住。其中费用自理的人员，招待所管理人员会开具行政事业单位统一的收据给他们，该收据是可以报销的。

3. 安排在研究生公寓的研究生：请找数学系张美蓉老师（13564296298）领取寝室的钥匙、电卡和空调遥控器。 研究生公寓入住须知如下：

（1）每间房间有：房门钥匙 2 把，阳台钥匙 1 把，电卡 1 张，空调遥控器 1 只。退房时，请以将这 5 件东西交至宿舍所在的 1 楼管理员处。如有缺少由入住人赔付。

（2）电卡充卡时间为：每天 12:00-16:00，请学生及时充卡，以免到时没电。提醒学生不要充太多，因为，退宿后卡里的钱是不能退的。

（3）入住研究生公寓学生名单：

序号	姓名	性别	入住日期	离开日期
1	张霞	女	7 月 27 日	8 月 9 日
2	王进	女	7 月 27 日	8 月 9 日
3	于冰	女	7 月 27 日	8 月 9 日
4	罗娟	女	7 月 27 日	8 月 9 日
5	孙平平	女	7 月 27 日	8 月 9 日
6	康晓丹	女	7 月 27 日	8 月 9 日
7	黄九龄	男	7 月 27 日	8 月 9 日
8	于志强	男	7 月 27 日	8 月 9 日
9	楼琦	男	7 月 27 日	8 月 8 日
10	朱瑞鹏	男	7 月 27 日	8 月 8 日

[致谢]