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A variational formulation for segmenting desired objects in color images

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Abstract

This paper presents a new variational formulation for detecting interior and exterior boundaries of desired object(s) in color images. The classical level set methods can handle changes in topology, but can not detect interior boundaries. The Chan–Vese model can detect the interior and exterior boundaries of all objects, but cannot detect the boundaries of desired object(s) only. Our method combines the advantages of both methods. In our algorithm, a discrimination function on whether a pixel belongs to the desired object(s) is given. We define a modified Chan–Vese functional and give the corresponding evolution equation. Our method also improves the classical level set method by adding a penalizing term in the energy functional so that the calculation of the signed distance function and re-initialization can be avoided. The initial curve and the stopping function are constructed based on that discrimination function. The initial curve locates near the boundaries of the desired object(s), and converges to the boundaries efficiently. In addition, our algorithm can be implemented by using only simple central difference scheme, and no upwind scheme is needed. This algorithm has been applied to real images with a fast and accurate result. The existence of the minimizer to the energy functional is proved in the Appendix A.
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1. Introduction

Image segmentation is one of the fundamental problems in image processing and computer vision. Among the image segmentation techniques, active contour methods grow significantly [3–5,9,11,19]. The basic idea in active contour methods is to evolve a curve, subject to a stopping function depending on the gradient of a gray image or the intensity of a color image. The active contour method can segment all objects in the image if the initial contour shrinks from the image boundary. However, it can only detect the exterior boundaries during the shrinkage. To overcome this problem, Chan and Vese proposed the method of active contours without edges (CV model) [6,7]. In the functional of CV model there are terms used to detect the boundary. These terms are based on Mumford–Shah segmentation techniques [14] and do not include a stopping function like active contour methods. The merit of the CV model is that it can detect both interior and exterior boundaries simultaneously wherever the initial curve starts. Furthermore, the boundaries detected by the CV model are not necessarily defined by the gradient. Based on these works, Kimmel [12] presented a unified framework of active contours. The energy functional combines the geodesic active contour model for regularization, the fitting energy from the CV model, and a term called the alignment as part of other driving forces from an active contour. The alignment means that the curves attain the boundaries when their normal best align with the image gradient.

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Kimmel's framework performs better than classical geodesic active contours when directional information about the edge location is provided.

All these methods are very efficient to segment all objects from a gray or color image. However, in some cases, only certain object(s) is/are the desired object(s) we wish to segment. In order to segment desired object(s) and regardless of other objects, many new models have been proposed recently [8,17,21]. They use a prior information such as shape or texture to distinguish the desired object(s) from others.

In this paper, we also focus on the segmentation of desired object(s) in color images, but we mainly make use of the color information, i.e. the Red, Green and Blue (RGB) information, of the desired object(s). Using the gradient information as well as the color information, we propose a variational formulation that can detect the interior and exterior boundaries of the desired object(s). The color information is used to construct a discrimination function that determines whether an image pixel belongs to the desired object(s) or not. The discrimination function is included in the energy functional and the corresponding evolution equation. With this discrimination function, the evolving curve will stop near the objects to be detected.

Our method also improves the classical level set method by avoiding the calculation of the signed distance function and re-initialization, inspired by the idea of Li et al. [13]. Computing signed distance function and re-initialization is an important step in classical let set method. To implement the classical level set method, we first define an initial curve, then set an initial level set function which is a signed distance to the initial curve [15]. A curve with simple shape, e.g. a circle or a rectangle, is often chosen to be the initial curve for the easy calculation of the distance function. Re-initialization is a remedy to keep the evolving level set function close to a signed distance function during the evolution [15,18]. However, it is hard to determine when and how the re-initialization should be applied. Thus, many proposed re-initialization schemes have an undesirable effect of moving the zero level set away from their original locations. We simplify the procedure of computing distance function and re-initialization by adding a penalizing term into the energy functional. Using our method, the initial curve is no longer necessarily restricted to be a curve with simple shape and is close to the boundaries of the desired object(s); and our initial level set function may not be a signed distance function, but a piecewise constant function. In order to force the level set function close to a signed distance function and avoid re-initialization, an internal energy term is added to the proposed variational formulation.

This paper is organized as follows. In Section 2, we review the classical geodesic active contour methods and the CV model. Section 3 describes our variational formulation based on the color information. The statistical methods of constructing the discrimination function are presented. Section 4 outlines the numerical method of the model. Experimental results are given in Section 5. Section 6 concludes the paper. The existence of the minimizer to the energy functional is proved in the Appendix A.

2. Classical geodesic active contour method and CV model

2.1. Geodesic active contours

Let Ω be a bounded open subset of $\mathbf{R}^2$, with $\partial\Omega$ its boundary. Let $I : \overline{\Omega} \to \mathbf{R}$ be a given gray level image, and $\vec{C}(p) = (x(p), y(p)),\ (p \in [0,1])$ be a differentiable parameterized curve in Ω. The geodesic active contour method is formulated by minimizing the energy functional:

$$E(\vec{C}) = \int_0^1 g(|\nabla I(\vec{C}(p))|)|\vec{C}'(p)|\,\mathrm{d}p$$

where $g(|\nabla I|)$ is called the stopping function. A typical stopping function can be chosen as

$$g(|\nabla I|) = \frac{1}{1 + K|\nabla(G_\sigma * I)|^2},$$

where $K > 0$ is a contract factor, $G_\sigma * I$ is the convolution of the image I with the Gaussian $G_\sigma(x,y) = \sigma^{-1/2}\exp(-|x^2 + y^2|/4\sigma)$, and σ is a parameter.

The stopping function $g(|\nabla I|)$ should be strictly positive in homogeneous regions and close to zero on the edges [4,5].

This stopping function should be modified in color images. For a color image $\vec{I} = (I_1, I_2, I_3) = (I_{\mathrm{R}}, I_{\mathrm{G}}, I_{\mathrm{B}})$, a new stopping function $g(x,y)$ is proposed in [9]:

$$g(x,y) = \frac{1}{1 + K\Lambda^2}, \tag{1}$$

where Λ is the largest eigenvalue of the structure tensor metric (g_{ij}) in the spatial–spectral space, and

$$(g_{ij}) = \begin{pmatrix} 1 + R_x^2 + G_x^2 + B_x^2 & R_xR_y + G_xG_y + B_xB_y \\ R_xR_y + G_xG_y + B_xB_y & 1 + R_y^2 + G_y^2 + B_y^2 \end{pmatrix}, \tag{2}$$

where R, G and B represent the pixel values of Red, Green and Blue after Gaussian convolution, respectively, i.e. $R = G_\sigma * I_1$, $G = G_\sigma * I_2$, $B = G_\sigma * I_3$ and $R_x = \partial_x R$ etc.

2.2. Active contours without edges

The model of active contours without edges (CV model) [7] is to segment all objects in a given image based on the Mumford–Shah model. Given a gray image, let the evolving curve $\vec{C}$ be the boundary of an open subset ϖ of Ω (i.e. $\varpi \subset \Omega$ and $\vec{C} = \partial\varpi$), inside($\vec{C}$) denotes the region ϖ, and outside($\vec{C}$) denotes the region $\Omega \setminus \bar{\varpi}$. Then, the energy functional is defined by:

$$E(c^+, c^-, \vec{C}) = \mu \cdot \text{Length}(\vec{C}) + v \cdot \text{Area}(\text{inside}(\vec{C})) + \lambda^+ \times \int_{\text{inside}(\vec{C})} |I(x,y) - c^+|^2 \, \mathrm{d}x\,\mathrm{d}y + \lambda^- \times \int_{\text{outside}(\vec{C})} |I(x,y) - c^-|^2 \, \mathrm{d}x\,\mathrm{d}y,$$

where $\mu \geqslant 0$, $v \geqslant 0$, λ^+, $\lambda^- > 0$ are fixed parameters, and c^+, c^- are two constants to be determined. In almost all cases, v is set to be zero. The first two terms are called regularizing terms, and the last two terms are called "fitting energy". Then, the minimization of this energy becomes:

$$\min_{c^+, c^-, \vec{C}} E(c^+, c^-, \vec{C}).$$

CV model can be generalized to vector-valued images [6]. Let I_i be the i-th channel of an image on Ω, $i = 1, \cdots, N$. The energy functional for the vector case is

$$E(\vec{c}^+, \vec{c}^-, \vec{C}) = \mu \cdot \text{Length}(\vec{C}) + \int_{\text{inside}(\vec{C})} \frac{1}{N} \sum_{i=1}^{N} \lambda_i^+ |I_i(x,y) - c_i^+|^2 \, \mathrm{d}x\,\mathrm{d}y + \int_{\text{outside}(\vec{C})} \frac{1}{N} \sum_{i=1}^{N} \lambda_i^- |I_i(x,y) - c_i^-|^2 \, \mathrm{d}x\,\mathrm{d}y,$$

where $\lambda_i^+ > 0$ and $\lambda_i^- > 0$ are parameters for each channel, and $\vec{c}^+ = (c_1^+, \cdots, c_N^+)$ and $\vec{c}^- = (c_1^-, \cdots, c_N^-)$ are two constant vectors to be determined.

3. Description of our model

Inspired by the advantages of both classical level set method and the CV model, we now present our variational approach for the detection of desired object(s) using color information.

3.1. Variational level set formulation for active contours

Let $\vec{I} = (I_1, I_2, I_3) = (I_R, I_G, I_B)$ be the Red, Green and Blue channels of a color image on Ω, and $\vec{C}$ the evolving curve. Let $\vec{c}^+ = (c_1^+, c_2^+, c_3^+)$ and $\vec{c}^- = (c_1^-, c_2^-, c_3^-)$ be two constant vectors to be determined. We first propose our energy functional as:

$$\begin{aligned} E(\vec{c}^+, \vec{c}^-, \vec{C}) = {} & \mu \int_0^1 g(\vec{C}(p)) |\vec{C}'(p)| \, \mathrm{d}p + v \int_\Omega g(\vec{C}(p)) \, \mathrm{d}A \\ & + \frac{1}{3} \int_{\text{inside}(\vec{C})} (G_\sigma * \alpha) \times \sum_{i=1}^{3} \lambda_i^+ |I_i(x,y) - c_i^+|^2 \, \mathrm{d}x\,\mathrm{d}y \\ & + \frac{1}{3} \int_{\text{outside}(\vec{C})} (G_\sigma * \alpha) \times \sum_{i=1}^{3} \lambda_i^- |I_i(x,y) - c_i^-|^2 \, \mathrm{d}x\,\mathrm{d}y, \end{aligned} \tag{3}$$

where $\lambda_i^+ > 0$, $\lambda_i^- > 0$, $\mu \geqslant 0$, $v \geqslant 0$ and Λ are defined as before, $g = \frac{1}{1+(G_\sigma * \alpha)\Lambda^2}$ is the stopping function, and α is a discrimination function which will be defined later in this paper. Similar to the CV model, the first two terms are the regularizing terms, and the last two terms are the modified "fitting energy" used to locate the interior and exterior boundaries of the desired object(s). Then, the minimization problem is

$$\min_{\vec{c}^+, \vec{c}^-, \vec{C}} E(\vec{c}^+, \vec{c}^-, \vec{C}). \tag{4}$$

We now re-write (3) so that numerical implementation of (4) can be conducted.

(3) can be formulated and solved using the level set method [16]. In the level set method, $\vec{C}$ is represented by the zero level set of a Lipschitz function ϕ: $\mathbf{R}^2 \to \mathbf{R}$, such that

$$\begin{cases} \vec{C} = \{(x,y) \in \mathbf{R}^2 : \phi(x,y) = 0\}; \\ \text{inside}(\vec{C}) = \{(x,y) \in \mathbf{R}^2 : \phi(x,y) > 0\}; \\ \text{outside}(\vec{C}) = \{(x,y) \in \mathbf{R}^2 : \phi(x,y) < 0\}. \end{cases}$$

Using the Heaviside function $H(z) = \begin{cases} 1, & \text{if } z \geqslant 0 \\ 0, & \text{if } z < 0 \end{cases}$ and the one-dimensional Dirac measure $\delta_0(z) = \frac{\mathrm{d}}{\mathrm{d}z} H(z)$ (in the sense of distributions) [22], (3) can be re-written as

$$\begin{aligned} E(\vec{c}^+, \vec{c}^-, \phi) = {} & \mu \int_\Omega g |DH(\phi(x,y))| + v \int_\Omega g H(\phi(x,y)) \, \mathrm{d}x\,\mathrm{d}y \\ & + \frac{1}{3} \int_\Omega (G_\sigma * \alpha) \sum_{i=1}^{3} \lambda_i^+ |I_i - c_i^+|^2 \times H(\phi(x,y)) \, \mathrm{d}x\,\mathrm{d}y \\ & + \frac{1}{3} \int_\Omega (G_\sigma * \alpha) \sum_{i=1}^{3} \lambda_i^- |I_i - c_i^-|^2 \times (1 - H(\phi(x,y))) \, \mathrm{d}x\,\mathrm{d}y, \end{aligned} \tag{5}$$

where $\int_\Omega g |DH(\phi(x,y))|$ is defined in Definition A.3 of the Appendix A.

Keeping ϕ fixed and minimizing (5) with respect to c_i^+ and c_i^-, $(i = 1, 2, 3)$, we obtain:

$$c_i^+ = \frac{\int_\Omega (G_\sigma * \alpha) I_i(x,y) H(\phi(x,y)) \, \mathrm{d}x\,\mathrm{d}y}{\int_\Omega (G_\sigma * \alpha) H(\phi(x,y)) \, \mathrm{d}x\,\mathrm{d}y} \quad (\text{average } (I_i) \text{ on } \phi \geqslant 0),$$

$$c_i^- = \frac{\int_\Omega (G_\sigma * \alpha) I_i(x,y) (1 - H(\phi(x,y))) \, \mathrm{d}x\,\mathrm{d}y}{\int_\Omega (G_\sigma * \alpha) (1 - H(\phi(x,y))) \, \mathrm{d}x\,\mathrm{d}y} \quad (\text{average } (I_i) \text{ on } \phi < 0).$$

Fixing c_i^+ and c_i^-, without loss of generality, we can denote $E(\vec{c}^+, \vec{c}^-, \phi)$ as $E(\phi)$. We need to calculate the first variation of $E(\phi)$ with respect to ϕ. To do so, we consider the slightly regularized versions of the functions H and δ_0. Let H_ε be any $C^2(\overline{\Omega})$ regularization of H, and $\delta_\varepsilon = H'_\varepsilon$ as $\varepsilon \to 0$. Here, we choose $H_\varepsilon(z) = \frac{1}{2}\left(1 + \frac{2}{\pi} \arctan\left(\frac{z}{\varepsilon}\right)\right)$.

The associated regularized functional E_ε is defined by:

$$\begin{aligned} E_\varepsilon(\vec{c}^+,\vec{c}^-,\phi) &= \mu\int_\Omega g(x,y)|\nabla H_\varepsilon(\phi(x,y))|\,\mathrm{d}x\,\mathrm{d}y \\ &\quad + \nu\int_\Omega g(x,y)H_\varepsilon(\phi(x,y))\,\mathrm{d}x\,\mathrm{d}y \\ &\quad + \frac{1}{3}\int_\Omega (G_\sigma * \alpha)\sum_{i=1}^{3}\lambda_i^+|I_i - c_i^+|^2 \\ &\quad \times H_\varepsilon(\phi(x,y))\,\mathrm{d}x\,\mathrm{d}y \\ &\quad + \frac{1}{3}\int_\Omega (G_\sigma * \alpha)\sum_{i=1}^{3}\lambda_i^-|I_i - c_i^-|^2 \\ &\quad \times (1 - H_\varepsilon(\phi(x,y)))\,\mathrm{d}x\,\mathrm{d}y. \end{aligned} \tag{6}$$

Similarly, when c_i^+ and c_i^- are fixed, we can denote $E_\varepsilon(\vec{c}^+,\vec{c}^-,\phi)$ as $E_\varepsilon(\phi)$. Minimizing (6) with respect to ϕ, we deduce the first variation of (6) for ϕ. Using the gradient method, the evolution equation (with the initial contour $\phi(0,x,y) = \phi_0(x,y)$) is

$$\begin{cases} \frac{\partial\phi}{\partial t} = \delta_\varepsilon(\phi)\Big[\mu\cdot\mathrm{div}\Big(g\frac{\nabla\phi}{|\nabla\phi|}\Big) - \nu\cdot g \\ \qquad - \frac{G_\sigma*\alpha}{3}\sum_{i=1}^{3}\lambda_i^+(I_i - c_i^+)^2 \\ \qquad + \frac{G_\sigma*\alpha}{3}\sum_{i=1}^{3}\lambda_i^-(I_i - c_i^-)^2\Big] \text{ in } (0,\infty)\times\Omega, \\ \frac{\delta_\varepsilon(\phi)}{|\nabla\phi|}\frac{\partial\phi}{\partial\vec{n}} = 0 \quad \text{on } \partial\Omega, \end{cases}$$

where $\vec{n}$ denotes the exterior normal to the boundary $\partial\Omega$.

Finally, we define the discrimination function α appeared in (3). The purpose of constructing such a discrimination function is to derive the characteristics of desired object(s) so that the characteristics of desired object(s) can be shown in the energy functional. This is done by analyzing n sample pixels chosen from the desired object(s). The Principal Components Analysis (PCA) and interval estimation are used in the analysis [1]. Using PCA, a new set of variables, called principal components [1], is obtained. Each principal component is a linear combination of the original variables. In our problem, at most the first two principal components are used. Using the interval estimation [1], we can define an interval that almost covers all samples for each principal component. Without loss of generality, we assume that only the first principal component is used. An interval $[a,b]$ for this principal component can be constructed by the interval estimation. When every pixel (x,y) of the color image is projected from its RGB values to the first principal component axis, we get a new value $\tilde{u}(x,y)$. If the value is within the interval, the pixel is probabilistically regarded as a pixel in the desired object(s). The discrimination function $\alpha(x,y)$ based on the color information is constructed as:

$$\alpha(x,y) = \begin{cases} 1 & a \leqslant \tilde{u}(x,y) \leqslant b; \\ 0 & \text{others.} \end{cases}$$

3.2. General variational level set formulation with penalizing energy

In order to keep the evolving level set function as an approximate signed distance function, we add a penalizing term $P(\phi) = \frac{1}{2}\int_\Omega(|\nabla\phi| - 1)^2\,\mathrm{d}x\,\mathrm{d}y$ into (5). Thus, our final energy functional becomes:

$$F(\phi) = \eta\cdot P(\phi) + E(\phi), \tag{7}$$

where $\eta > 0$ is a parameter controlling the effect of penalizing term, and $E(\phi)$ is defined in (5). $P(\phi)$ describes the deviation of level set function ϕ from the signed distance function. Thus, the evolving level set function for (7) keeps as an approximate signed distance function during the evolution [2]. The existence of the minimizer to the energy functional (7) is proved in Appendix A.

In practical computing, the following energy functional is used:

$$F_\varepsilon(\phi) = \eta\cdot P(\phi) + E_\varepsilon(\phi).$$

The gradient method is used to compute the minimizer of $F_\varepsilon(\phi)$. The new evolution equation is:

$$\begin{cases} c_i^+(\phi) = \dfrac{\int_\Omega (G_\sigma*\alpha)I_i(x,y)H_\varepsilon(\phi(x,y))\,\mathrm{d}x\,\mathrm{d}y}{\int_\Omega (G_\sigma*\alpha)H_\varepsilon(\phi(x,y))\,\mathrm{d}x\,\mathrm{d}y}, & \text{(8a)} \\ c_i^-(\phi) = \dfrac{\int_\Omega (G_\sigma*\alpha)I_i(x,y)(1 - H_\varepsilon(\phi(x,y)))\,\mathrm{d}x\,\mathrm{d}y}{\int_\Omega (G_\sigma*\alpha)(1 - H_\varepsilon(\phi(x,y)))\,\mathrm{d}x\,\mathrm{d}y}, & \text{(8b)} \\ \dfrac{\partial\phi}{\partial t} = \eta\cdot\mathrm{div}\left[\left(1 - \dfrac{1}{|\nabla\phi|}\right)\nabla\phi\right] & \\ \qquad + \delta_\varepsilon(\phi)\Big[\mu\cdot\mathrm{div}\Big(g\dfrac{\nabla\phi}{|\nabla\phi|}\Big) & \\ \qquad - \nu\cdot g - \dfrac{G_\sigma*\alpha}{3}\sum_{i=1}^{3}\lambda_i^+(I_i - c_i^+)^2 & \\ \qquad + \dfrac{G_\sigma*\alpha}{3}\sum_{i=1}^{3}\lambda_i^-(I_i - c_i^-)^2\Big] \text{ in } (0,\infty)\times\Omega, & \text{(8c)} \\ \phi(0,x,y) = \phi_0(x,y) \text{ in } \Omega, & \text{(8d)} \\ \dfrac{\delta_\varepsilon(\phi)}{|\nabla\phi|}\dfrac{\partial\phi}{\partial\vec{n}} = 0 \text{ on } \partial\Omega. & \text{(8e)} \end{cases}$$

4. Numerical approximation of the model

The discrete algorithm for (8a)–(8e) is: knowing ϕ^n, we first compute $c_i^+(\phi^n)$ and $c_i^-(\phi^n)$ $(i = 1,2,3)$. All the spatial partial derivatives $\frac{\partial\phi}{\partial x}$ and $\frac{\partial\phi}{\partial y}$ can be approximated by the central differences, since the upwind scheme [20] in the classical level set methods is no longer needed due to the diffusion properties of the penalizing energy. Then, ϕ^{n+1} is given by the following discrete scheme:

$$\phi_{i,j}^{n+1} = \phi_{i,j}^n + \tau\cdot L(\phi_{i,j}^n), \tag{9}$$

where $L(\phi_{i,j}^n)$ is the approximation of the right-hand side in (8c).

Here, the initial function ϕ_0 is defined as:

$$\phi_0(x,y) = \rho \cdot \left(G_\sigma * \alpha(x,y) - \frac{1}{2} \right), \tag{10}$$

where $\rho > 0$ is a constant. We suggest that ρ is set to be larger than 2ε, where ε is the width in $H_\varepsilon(z) = \frac{1}{2}\left(1 + \frac{2}{\pi}\arctan\left(\frac{z}{\varepsilon}\right)\right)$. The initial curve $\phi_0(x,y) = 0$ is close to the boundaries of the desired object(s), guaranteed by the definition of the discrimination function.

The main steps of the algorithm can be summarized as:

1. choose sample pixels in the desired object(s);
2. construct the discrimination function α by the PCA and interval estimation;
3. initialize $\phi^n(n = 0)$ by (10);
4. compute $c_i^+(\phi^n)$ and $c_i^-(\phi^n)$ $(i = 1, 2, 3)$ using (8a) and (8b);
5. compute ϕ^{n+1} by (9);
6. check whether the solution is stationary. If not, $n = n + 1$ and repeat from 4.

5. Experimental results

The proposed variational formulation has been applied to real color images and the experimental results are accurate. Here, three examples are demonstrated. The algorithm is implemented by Matlab on a Pentium 1.86 GHz computer with 512 M memory. The processing time referred later in the paper starts after choosing the sample pixels. In each experiment, the level set function is initialized by the function ϕ_0 defined in (10) with $\rho = 4$, and the width ε in H_ε is $\varepsilon = 1$.

We first consider a simple case. Fig. 1(a) shows a 364×400-pixel image of two different bags with complicated textured background. Our desired object is the blue bag. The classical geodesic active contour method for color images fails since some noises are mis-detected as the boundaries of objects. The CV model can segment all the objects since it is noise resistant, but the result contains both bags and not the blue bag that we desire. Nine sample pixels are chosen from the blue bag. By the initial level set function ϕ_0, the initial curve is very close to the boundaries of the desired object. The curve evolution only takes 10 iterations to reach the interior and exterior boundaries of the blue bag (see Fig. 1(b)). The processing time is 6.67 s.

The second example shows the ability of our algorithm to a real case. Fig. 2(a) shows a 472×268-pixel nature color image. This image is a part of a maidenhair tree. Our purpose is to segment the leaves, ignoring both the trunk and the background. Nine sample pixels are chosen from the leaves. Fig. 2(b) shows the location of the initial curve. The curve evolution takes 75 iterations (see Fig. 2(c)). The processing time is 27.41 s.

The third example is used to demonstrate the robustness of our method when the object boundaries are complicated. Fig. 3(a) shows a 350×262-pixel nature color image. Seven sample pixels are chosen from the yellow leaves. It takes five iterations to reach the desired object (see Fig. 3(b)). The processing time is 5.64 s.

6. Conclusions

In this paper, we have presented a new variational formulation for active contours. This method is a generalization of

Fig. 1. Result for the detection of the blue bag. $\tau = 5$, $\sigma = 0.6$, $\mu = 1$, $\lambda^+ = (0.5, 0.5, 0.5)$, $\lambda^- = (1, 1, 1)$.

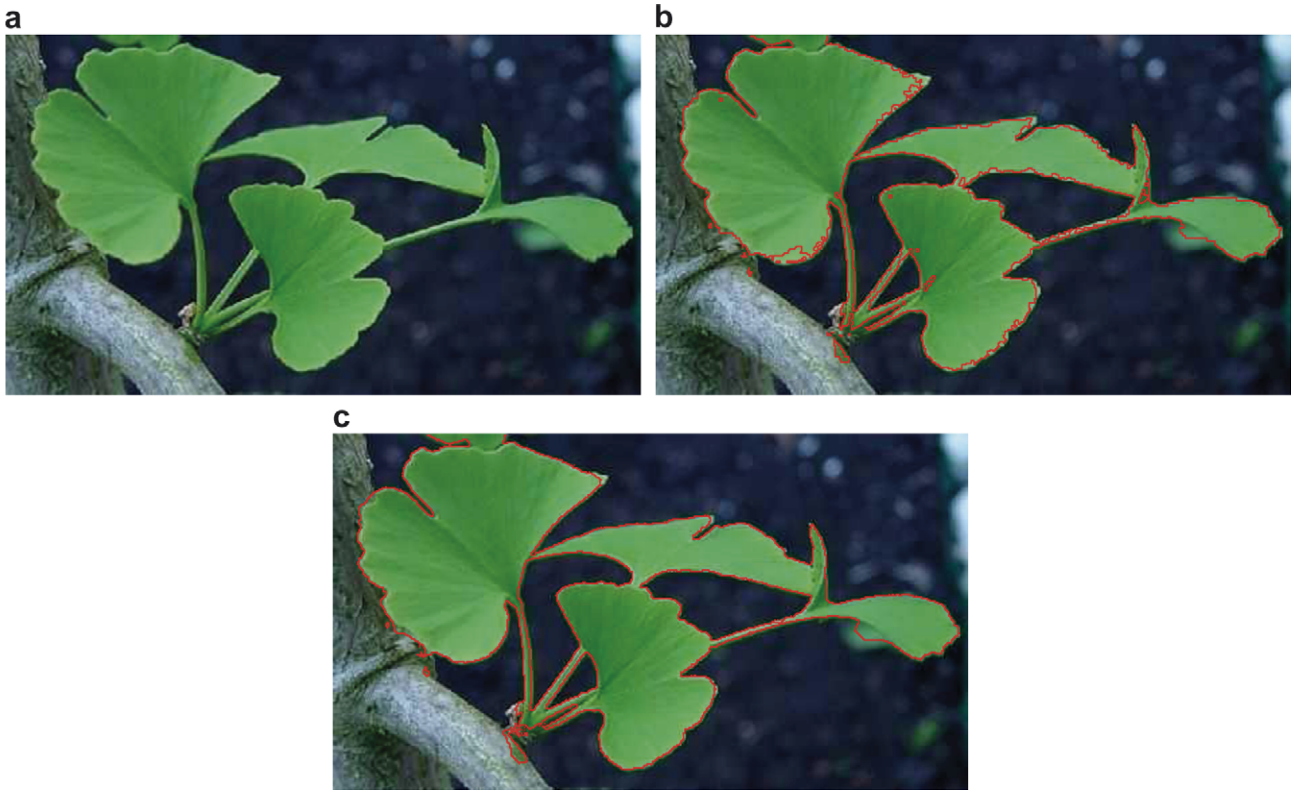

Fig. 2. Result for the detection of maidenhair leaves. $\tau = 5$, and $\eta = 0.00001$, $\sigma = 0.5$, $\mu = 1$, $\nu = 0.5$, $\lambda^{+} = (0.35, 0.35, 0.35)$, $\lambda^{-} = (1, 1, 1)$.

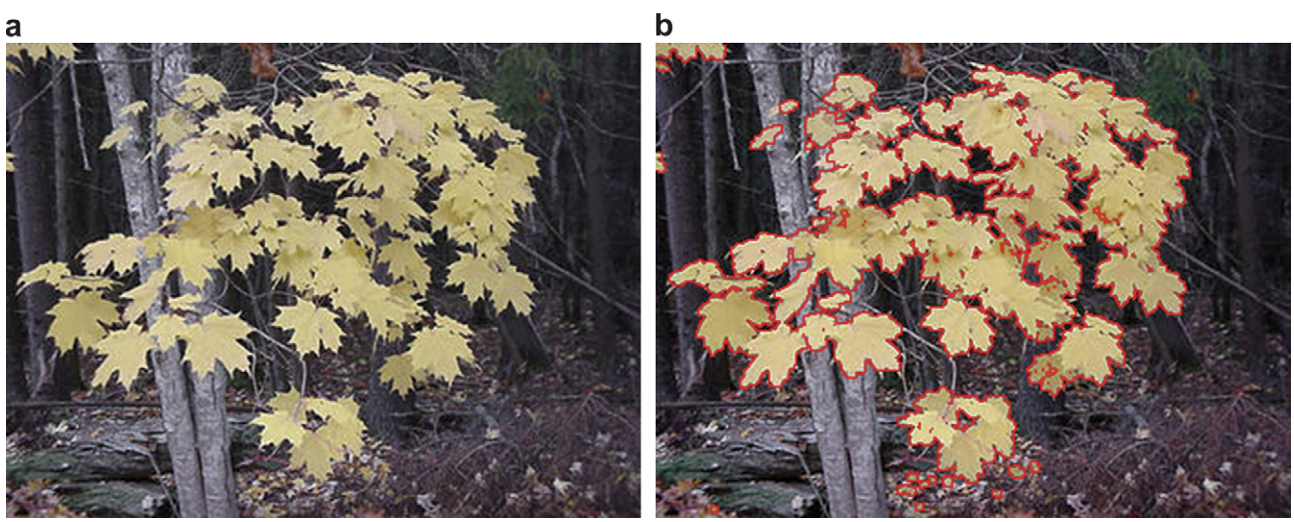

Fig. 3. Result for the detection of the complicated object boundaries. $\tau = 5$, and $\sigma = 0.8$, $\eta = 0.00001$, $\mu = 0.01$, $\lambda^{+} = (0.3, 0.3, 0.3)$, $\lambda^{-} = (1, 1, 1)$.

the classical geodesic active contour method and the CV model. It can simultaneously detect the interior and exterior boundaries of desired object(s) in color images, regardless of other objects. Moreover, in the implementation of the level set method, we have applied Li's method [13] to our case which is detecting desired object(s) only. The calculation of the signed distance function and re-initialization are avoided by adding a penalizing term into the energy functional. The initial level set function is set automatically and is close enough to the boundaries of the desired object(s). The penalizing term forces the evolving level set functions to be close to signed distance functions during the evolution. In addition, our algorithm can be implemented by using only simple central difference schemes and is more computationally efficient than the classical level set methods.

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Appendix A. Existence of the minimizer

In this Appendix A, we shall prove the existence of the solution to the minimization in (7). As a preparation, we first prove the existence of the solution to the minimization in (3).

Because c_i^+ and c_i^- $(i=1,2,3)$ have explicit representations as functions of ϕ, we can consider the energy in ϕ only: $E(\phi,\vec{c}^+(\phi),\vec{c}^-(\phi))$. In this case, $E(\phi,\vec{c}^+(\phi),\vec{c}^-(\phi))$ is expressed as a function of $H(\phi)$ only, where $H(\phi)$ is a characteristic function. Let $K=\{(x,y)\in\Omega|\phi(x,y)\geqslant 0\}$, and χ_K be the characteristic function of K, so $\chi_K=H(\phi)$. Therefore, as suggested in [7], (3) can be rewritten in the form:

$$\min_{\chi_K} E(\chi_K)=\min_{\chi_K}\Big\{\mu\int_\Omega g|D\chi_K|+v\int_\Omega g\chi_K\,dx\,dy + \frac{1}{3}\int_\Omega \beta\sum_{i=1}^{3}\lambda_i^+(I_i-c_i^+(\chi_K))^2\chi_K\,dx\,dy + \frac{1}{3}\int_\Omega \beta\sum_{i=1}^{3}\lambda_i^-(I_i-c_i^-(\chi_K))^2(1-\chi_K)\,dx\,dy\Big\}, \tag{A1}$$

where $\beta=G_\sigma*\alpha$ is a smooth bounded function, $0\leqslant\beta\leqslant 1$, g is also smooth. This minimization is over all the characteristic functions of K in $BV(\Omega)$ (note that the set K varies as ϕ evolves).

In order to study the existence for the problem (A1), it is necessary to introduce the concept of weighted total variation norms for functions of bounded variation. We first recall some definitions about the functions with bounded variation [10,23].

Definition A.1. Let $\Omega\subset\mathbf{R}^N$ be an open, bounded and connected set, and $u\in L^1(\Omega)$. Define

$$\int_\Omega |Du| = \sup_f\Big\{\int_\Omega u\ \mathrm{div} f\,dx \Big| f\in C_c^1(\Omega;\mathbf{R}^N), |f|\leqslant 1\Big\}.$$

Definition A.2. A function $u\in L^1(\Omega)$ is said to have bounded variation in Ω, if $\int_\Omega |Du|<\infty$. We denote it by $u\in BV(\Omega)$. The norm of u in this Banach space is given by

$$\|u\|_{BV(\Omega)}=\|u\|_{L^1(\Omega)}+\int_\Omega |Du|.$$

Next we define the norm of weighted total variation with the weight function $\gamma(x)$.

Definition A.3. Let $\Omega\subset\mathbf{R}^N$ be an open, bounded and connected set, and $u\in L^1(\Omega)$. Let $\gamma(x)$ be a nonnegative continuous and bounded function on Ω. The weighted total variation norm of u with the weight function $\gamma(x)$, denoted by $\int_\Omega\gamma(x)|Du|$, is defined by

$$\int_\Omega \gamma(x)|Du| = \sup_f\Big\{\int_\Omega u\ \mathrm{div} f\,dx \Big| f\in C_c^1(\Omega;\mathbf{R}^N), |f|\leqslant \gamma(x)\Big\}.$$

Lemma A.4. *Assume that the perimeter of a subset K in Ω is defined by $\mathrm{per}_\Omega(K)=\int_\Omega |D\chi_K|$, then K has finite perimeter if and only if the characteristic function $\chi_K\in BV(\Omega)$.*

It is well known that if $u\in BV(\Omega)$, then for any $t\in\mathbf{R}$, the level set $K_t=\{x\in\Omega — u(x)>t\}$ has finite perimeter, i.e. $\chi_{K_t}\in BV(\Omega)$.

Lemma A.5. [10] *Let $\Omega\subset\mathbf{R}^N$ be an open set with a Lipschitz boundary. Suppose that $\{u_n\}$ is a bounded sequence in $BV(\Omega)$. Then, there is a subsequence $\{u_{n_j}\}$ converging strongly in $L^p(\Omega)$ to $u\in BV(\Omega)$ for any $1\leqslant p<\frac{n}{n-1}$.*

Main Theorem. *Let $\Omega\subset\mathbf{R}^2$. If $I_i\in L^\infty(\Omega)$, $(i=1,2,3)$, then the minimization problem (A1) has a solution $\chi_K\in BV(\Omega)$.*

Proof. Let $\{\chi_{K_n}\}$, $n\geqslant 1$ be a minimizing sequence of (A1), i.e.
$\min_{\chi_K} E(\chi_K)=\lim_{n\to\infty}E(\chi_{K_n})$.

In our case, $g=\frac{1}{1+(G_\sigma*\alpha)\Lambda^2}$, where Λ is the largest eigenvalue of (2), and the structure tensor metric (2) is symmetric positive. Thus,

$$\begin{aligned}\Lambda &\leqslant \mathrm{trace}(g_{ij})\\ &= 2+R_x^2+R_y^2+G_x^2+G_y^2+B_x^2+B_y^2\\ &= 2+|\nabla(G_\sigma*I_1)|^2+|\nabla(G_\sigma*I_2)|^2+|\nabla(G_\sigma*I_3)|^2\\ &= 2+|\nabla G_\sigma*I_1|^2+|\nabla G_\sigma*I_2|^2+|\nabla G_\sigma*I_3|^2\\ &\leqslant 2+C_1\|I_1\|^2_{L^\infty(\Omega)}+C_2\|I_2\|^2_{L^\infty(\Omega)}+C_3\|I_3\|^2_{L^\infty(\Omega)},\end{aligned}$$

where $C_i=C_i(\sigma)>0$, $(i=1,2,3)$. Since $I_i\in L^\infty(\Omega)$, $(i=1,2,3)$, there exists a constant $C(\sigma,I_1,I_2,I_3)>0$ such that $\frac{1}{1+C}\leqslant g(x,y)\leqslant 1$. So $g(x,y)$ has a positive lower bound. Then from the first term of the right hand of (A1), there is a constant $M>0$ such that $\int_\Omega |D\chi_{K_n}|\leqslant M$, for all $n\geqslant 1$. Therefore, χ_{K_n} is a bounded sequence in $BV(\Omega)$. By Lemma A.5, there exist a subsequence $\{\chi_{K_{n_i}}\}$ of $\{\chi_{K_n}\}$ and $u\in BV(\Omega)$ such that $\chi_{K_{n_j}}\to u$ strongly in $L^1(\Omega)$ and $\chi_{K_{n_j}}\to u$ a.e. on Ω. u is either 0 or 1 a.e. on Ω since χ_{K_n} is either 0 or 1. Thus, u can be viewed as the characteristic function χ_K of a set K which has finite perimeter in Ω.

Define $\mathscr{F}_g=\{f\in C_c^1(\Omega;\mathbf{R}^2)\,|\,|f(x,y)|\leqslant g(x,y)$ on $\Omega\}$. It is clear that

$$\int_\Omega \chi_{K_{n_j}} \operatorname{div} f \,\mathrm{d}x\,\mathrm{d}y \leqslant \sup_{f\in\mathcal{F}_g} \int_\Omega \chi_{K_{n_j}} \operatorname{div} f \,\mathrm{d}x\,\mathrm{d}y = \int_\Omega g|\mathrm{D}\chi_{K_{n_j}}|. \quad \text{(A2)}$$

Furthermore, since $\chi_{K_{n_j}} \to u$ a.e. on Ω, by dominated convergence theorem, we have

$$\int_\Omega \chi_K \operatorname{div} f \,\mathrm{d}x\,\mathrm{d}y = \lim_{n_j\to\infty} \int_\Omega \chi_{K_{n_j}} \operatorname{div} f \,\mathrm{d}x\,\mathrm{d}y.$$

By (A2), we take the inferior limit to get

$$\int_\Omega \chi_K \operatorname{div} f \,\mathrm{d}x\,\mathrm{d}y = \liminf_{n_j\to\infty} \int_\Omega \chi_{K_{n_j}} \operatorname{div} f \,\mathrm{d}x\,\mathrm{d}y \leqslant \liminf_{n_j\to\infty} \int_\Omega g|\mathrm{D}\chi_{K_{n_j}}|.$$

Therefore,

$$\sup_{f\in\mathcal{F}_g} \int_\Omega \chi_K \operatorname{div} f \,\mathrm{d}x\mathrm{d}y \leqslant \liminf_{n_j\to\infty} \int_\Omega g|\mathrm{D}\chi_{K_{n_j}}|.$$

Using Definition A.3, we have

$$\int_\Omega g|\mathrm{D}\chi_K| \leqslant \liminf_{n_j\to\infty} \int_\Omega g|\mathrm{D}\chi_{K_{n_j}}|.$$

The continuity or lower semi-continuity in $L^1(\Omega)$ of the fitting energy and $\int_\Omega g\cdot\chi_K \,\mathrm{d}x\,\mathrm{d}y$ can be obtained since β is smooth and bounded, $I_i \in L^\infty(\Omega)$, $(i=1,2,3)$, and $\chi_{K_{n_j}}$ is either 0 or 1.

In summary, by the lower semi-continuity of the total variation and the strong convergence in $L^1(\Omega)$ of the other terms, we have:

$$E(\chi_K) \leqslant \liminf_{n_j\to\infty} E(\chi_{K_{n_j}}).$$

Then χ_K is a minimizer of E among the characteristic functions of subsets with finite perimeter in Ω. □

Now, we shall prove the existence of the minimizer of the total energy (7): $F(\phi) = \eta\cdot P(\phi) + E(\phi)$. Let $\partial K = \{(x,y)\in\Omega|\phi(x,y)=0\}$, where $K=\{(x,y)\in\Omega|\phi(x,y)\geqslant 0\}$. Since any function ϕ on Ω can be characterized by the set $\{K, \phi^+(x,y), \phi^-(x,y)\}$, where K is the zero set of function ϕ, positive function $\phi^+(x,y)$ is defined on K and negative function $\phi^-(x,y)$ is defined on $\Omega\backslash K$. By the previous discussion, $E(\phi) = E(\chi_K)$. Thus, $E(\phi)$ is only dependent on the choice of set K. In general, $\min_\phi F(\phi) \geqslant \min_\phi E(\phi)$. However, $P(\phi)=0$ can be reached. As shown in [20], if K is fixed, we can choose a ϕ^+ on K as a positive distance function to ∂K, and a ϕ^- on $\Omega\backslash K$ as a negative distance function to ∂K, thus $|\nabla\phi| = 1$ on the whole domain Ω and $P(\phi)=0$. So the infimum of the total energy (7) is the same as the infimum of the energy (3). Therefore, we have proved the existence of the minimizer of (7) when the existence of the minimizer of (3) is proved.

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