Computers and Electrical Engineering 37 (2011) 782–788

Contents lists available at ScienceDirect

Computers and Electrical Engineering

journal homepage: www.elsevier.com/locate/compeleceng

# A fast implementation algorithm of TV inpainting model based on operator splitting method ☆

Fang Li [a], Chaomin Shen [b], Ruihua Liu [c], Jinsong Fan [d,*]

[a] Department of Mathematics, East China Normal University, Shanghai, China
[b] Department of Computer Science, East China Normal University, Shanghai, China
[c] School of Mathematics and Statistics, Chongqing University of Technology, Chongqing, China
[d] College of Mathematics and Information Science, Wenzhou University, Zhejiang, China

ARTICLE INFO

*Article history:*
Available online 23 June 2011

ABSTRACT

In this paper, we propose a fast algorithm to solve the well known total variation (TV) inpainting model. Classically, the Euler–Lagrange equation deduced from TV inpainting model is solved by the gradient descent method and discretized by an explicit scheme, which produces a slow inpainting process. Sometimes an implicit scheme is also used to tackle the problem. Although the implicit scheme is several times faster than the explicit one, it is still too slow in many practical applications. In this paper, we propose to use an operator splitting method by adding new variables in the Euler–Lagrange equation of TV inpainting model such that the equation is split into a few very simple subproblems. Then we solve these subproblems by an alternate iteration. Numerically, the proposed algorithm is very easy to implement. In the numerical experiments, we mainly compare our algorithm with the existing implicit TV inpainting algorithms. It is shown that our algorithm is about ten to twenty times faster than the implicit TV inpainting algorithms with similar inpainting quality. The comparison of our algorithm with harmonic inpainting algorithm also shows some advantages and disadvantages of the TV inpainting model.



## 1. Introduction

Image inpainting is an important topic in image processing. The goal of inpainting is to recover missing data in a damaged image. Its applications cover film restoration, text removal, scratch removal, and special effects in movies. An important class of digital inpainting methods relates to partial differential equations (PDEs). The term of *digital inpainting* was initially introduced by Bertalmío et al. [1]. The authors were also the first to apply PDEs in image inpainting. They proposed a third-order gradient descent flow to propagate the information outside the inpainting region into the inpainting region along isophotes. Ballester et al. [2] proposed to smoothly extend inside the inpainting domain both the vector field obtained from the image gradient and the isophotes. Bertalmío et al. [3] introduced the Navier–Stokes equations for fluid dynamics into image inpainting. Chan et al. [4] proposed total variation (TV) inpainting algorithm by modifying the Rudin–Osher–Fatemi (ROF) model [5]. Chan et al. gives the error analysis of TV inpainting model in Chan and Kang [6]. In [7], the same authors of [4] proposed a curvature-driven diffusion (CDD) PDE inpainting model which extends the TV algorithm by taking into account geometric information of isophotes (i.e. curvature) in the total variation diffusion equation, thus can connect some broken edges and maintain curvature of isophotes. Masnou et al. [8] and Chan et al. [9] studied the variational inpainting models based on Euler elastica in which curvature is also involved. Grossauer et al. used the complex Ginzburg Landau equation for digital inpainting

☆ Reviews processed and approved for publication to the Editor-in-Chief Dr. Manu Malek.

* Corresponding author.
*E-mail address:* fjs@wzu.edu.cn (J. Fan).

0045-7906/$ - see front matter © 2011 Elsevier Ltd. All rights reserved.
doi:10.1016/j.compeleceng.2011.05.014

in 2D and 3D [10]. Tai et al. [11] proposed to first propagate the isophote directions into the inpainting region by TV-Stokes equation and then restore the image by fitting the constructed directions. These PDE-based image inpainting methods have the advantage of preserving edges very well, however, the slow and unstable numerical implementations greatly limit their practical use. PDE based inpainting techniques are suitable for non-texture image inpainting.

Another class of approaches based on texture synthesis is suitable for texture image inpainting, in which the exemplar-based method seems to be very successful [12–14]. The two classes of methods can be combined together [15,16]. Variational nonlocal method is also proved to be successful in texture inpainting [17].

All of the above works considered inpainting in image domain. Recently, there were some works which considered inpainting in transformed domain. For instance, You et al. [18] studied the problem in wavelet domain, and Cai et al. [19] studied in both the image domain and general transformed domain.

In this paper we implement the TV inpainting model [4] by the operator splitting method using the technique developed in Shen [20]. Since it is hard to give the iteration formula for the inpainted image directly, we use the operator splitting method [21,22] after obtaining the Euler–Lagrange equation. This is done by introducing intermediate variables, and each variable can be solved relatively easier. Thus we could establish an alternate iteration formula so that the inpainting process could be expressed by the iteration. The algorithm is fast and simple to implement.

The outline of this paper is as follows. In Section 2, we propose our method to solve TV inpainting problem for gray scale images and color images respectively. In Section 3, we give the numerical results of our algorithm. We also compare our approach with the implicit TV inpainting algorithm in Chan and Shen [4]. Finally, we conclude the paper in Section 4.

## 2. The proposed method

### 2.1. TV inpainting model

Assume $\Omega \subset \mathbb{R}^2$ be the image domain and $u_0 : \Omega \rightarrow \mathbb{R}$ be the gray scale image. Let $D \subset \Omega$ be the inpainting domain and $D^c = \Omega \setminus D$ be the complement of $D$ in $\Omega$. The TV inpainting model proposed in [4] is a modified ROF model as

$$\min_{u(x)\in BV(\Omega)} E(u(x)) = TV(u(x)) + \frac{1}{2}\int_\Omega \hat{\lambda}(x)(u(x)-u_0(x))^2dx \tag{1}$$

where

$$\hat{\lambda}(x) = \begin{cases} \lambda, x \in D^c \\ 0, x \in D \end{cases}$$

and $\lambda$ is a positive constant. Recall that there are two definitions of TV norm as:

$$\text{anisotropic} \quad TV(u) = \|\nabla_{x_1}u\|_1 + \|\nabla_{x_2}u\|_1,$$

$$\text{isotropic} \quad TV(u) = \left\|\sqrt{\nabla_{x_1}u^2 + \nabla_{x_2}u^2}\right\|_1,$$

where $x = (x_1, x_2) \in \Omega$ and $\|\cdot\|_1$ denotes the $L^1$ norm. Remark that the isotropic one is always used in image inpainting.

The above TV inpainting model is classically solved by the gradient descent method and discretized by an explicit scheme or implicit scheme which is relatively slow. Though many fast algorithms (such as Chambolle's fast dual projection method [23] and the split Bregman method [24]) have been developed to solve the ROF model, they cannot be directly used in TV inpainting since $\hat{\lambda}$ takes zero value in the inpainting regions. In this paper, we will propose a new fast algorithm to solve TV inpainting problem.

### 2.2. The proposed algorithm

Consider the following TV model

$$\min_{u\in BV(\Omega)} E(u) = \alpha TV(u) + \frac{1}{2}\int_\Omega \hat{\lambda}(u-u_0)^2dx \tag{2}$$

where $\alpha$ is a positive weight parameter. For ease of notation, let $L = \nabla = (\nabla_{x_1}, \nabla_{x_2})$ denote the gradient operator along the first and second coordinates, $f$ is an operator such that $(TV(u) = f(Lu)$. Note that the two definitions of TV norm correspond to two definitions of the operator $f$.

By variational method, we get the Euler–Lagrange equation of Eq. (2)

$$0 \in \alpha\partial(f \circ L)(u) + \hat{\lambda}(u-u_0) \tag{3}$$

where $\partial$ denotes the subdifferential operator. Eq. (3) is equivalent to

$$0 \in L^*\partial f(\alpha Lu) + \hat{\lambda}(u-u_0) \tag{4}$$

Using the general property:

$$x \in \partial f(y) \Longleftrightarrow y \in \partial f^*(x) \tag{5}$$

for a convex function $f$ and its convex conjugate

$$f^*(y) = \sup_x \{\langle y, x\rangle - f(x)\}$$

we obtain the equivalent condition: $u$ satisfies Eq. (4) if and only if there exists an auxiliary variable $\mathbf{w} = (w_1, w_2)$ such that

$$0 = L^*\mathbf{w} + \widehat{\lambda}(u - u_0) \tag{6}$$

$$\alpha Lu \in \partial f^*(\mathbf{w}) \tag{7}$$

where $L^*$ is the adjoint operator of $L$. To solve the Eqs. (6) and (7), we use the operator splitting method [21,22] by introducing two scalars $\tau_1, \tau_2 > 0$, and obtain

$$(6) \Longleftrightarrow 0 = \tau_1 L^*\mathbf{w} + u - s \tag{8}$$

$$s = u - \widehat{\lambda}\tau_1(u - u_0) \tag{9}$$

$$(7) \Longleftrightarrow \mathbf{0} \in \tau_2 \partial f^*(\mathbf{w}) + \mathbf{w} - \mathbf{t} \tag{10}$$

$$\mathbf{t} = \mathbf{w} + \tau_2 \alpha Lu \tag{11}$$

From Eq. (8), it is easy to get the solution of $u$

$$u = s - \tau_1 L^*\mathbf{w} \tag{12}$$

Due to Eq. (5), Eq. (10) is equivalent to

$$\mathbf{0} \in \tau_2 \mathbf{w} + \partial f(\mathbf{w} - \mathbf{t})$$

Using the definition of $f$ which is determined by the definition of TV norm, we can deduce that the solution is

$$\mathbf{w} = \min\left\{\|\mathbf{t}\|, \frac{1}{\tau_2}\right\}\frac{\mathbf{t}}{\|\mathbf{t}\|} \tag{13}$$

where $\frac{\mathbf{0}}{0}$ is defined to be $\mathbf{0}$, $\|\mathbf{t}\| = \sqrt{t_1^2 + t_2^2}$ for the isotropic TV norm and $\|\mathbf{t}\| = |t_1| + |t_2|$ for the anisotropic TV norm. See [22] for the derivation of the solution for the isotropic case. The anisotropic case can be solved in a similar way, which is thus omitted.

In the discrete case, assume the image is an $n \times n$ matrix, then the operators $\nabla = (\nabla_{x_1}, \nabla_{x_2})$ are defined as

$$\begin{aligned}
&\nabla_{x_1} u(1,j) = 0, j = 1:n\\
&\nabla_{x_1} u(i,j) = u(i,j) - u(i-1,j), i = 2:n, j = 1:n\\
&\nabla_{x_2} u(i,1) = 0, i = 1:n\\
&\nabla_{x_2} u(i,j) = u(i,j) - u(i,j-1), i = 1:n, j = 2:n.
\end{aligned}$$

It is easy to derive that the adjoint operators of $\nabla = (\nabla_{x_1}, \nabla_{x_2})$ are $\nabla^T = (\nabla_{x_1}^T, \nabla_{x_2}^T)$ defined as

$$\begin{aligned}
&\nabla_{x_1}^T u(n,j) = 0, j = 1:n\\
&\nabla_{x_1}^T u(i,j) = u(i,j) - u(i+1,j), i = 1:n-1, j = 1:n\\
&\nabla_{x_2}^T u(i,n) = 0, i = 1:n\\
&\nabla_{x_2}^T u(i,j) = u(i,j) - u(i,j+1), i = 1:n, j = 1:n-1
\end{aligned}$$

and $L^*\mathbf{w} = \nabla^T\mathbf{w} = \nabla_{x_1}^T w_1 + \nabla_{x_2}^T w_2$.

Hence, we propose to solve the TV inpainting problem by the following alternative iteration algorithm.

*The algorithm*

- Initialization: $u^0 = u_0, \mathbf{w}^0 = \mathbf{0}$.
- Iteration: for $k$=0, 1,2,...

$$\begin{aligned}
&s^{k+1} = u^k - \widehat{\lambda}\tau_1(u^k - u_0)\\
&u^{k+1} = s^{k+1} - \tau_1 \nabla^T \mathbf{w}^k\\
&\mathbf{t}^{k+1} = \mathbf{w}^k + \tau_2 \alpha \nabla u^k\\
&\mathbf{w}^{k+1} = \min\left\{\|\mathbf{t}^{k+1}\|, \frac{1}{\tau_2}\right\}\frac{\mathbf{t}^{k+1}}{\|\mathbf{t}^{k+1}\|}
\end{aligned}$$

- Termination criterion: $k > k_{\max}$ where $k_{\max}$ is the maximum iteration defined by the user.

It is straightforward to extend the above algorithm into color image case. We omit the details.

## 3. Numerical results

We test the proposed algorithm on various images in this section. Default parameters are $\lambda = 10 * (1 - \chi_D)$, $\tau_1 = 0.1, \tau_2 = 0.03$ and we choose parameter $\alpha$ by letting $\tau_2\alpha = 1$. The default TV norm is isotropic. If other parameters are used, we will point out. The experiments are performed under Windows XP and MATLAB v7.4 with Intel Core 2 Duo CPU at 1.66 GHz and 2 GB memory.

### 3.1. Test on gray images

We test our algorithm for gray scale image inpainting in Figs. 1–3 . Some of the results are compared with TV inpainting algorithm in [4]. In Fig. 1, we show the intermediate results in the evolution process of our algorithm in the first row. The result at iteration 200 consuming 4.2 s seems quite good. In contrast, we show the results of TV inpainting method [4] (implicit method) in the evolution process in the second row. The result at iteration 5000 consuming 87.4 s is acceptable. Our algorithm is obviously faster than that in [4].

In Fig. 2, we test our algorithm using Lena image with 30% and 10% information left which is chosen randomly. The inpainted images by the proposed algorithm are satisfactory.

In Fig. 3, three synthetic images are tested by the proposed algorithms with anisotropic/isotropic TV norm and the harmonic inpainting algorithm [25]. The harmonic inpainting algorithm is aimed to solve the Laplace equation with Dirichlet

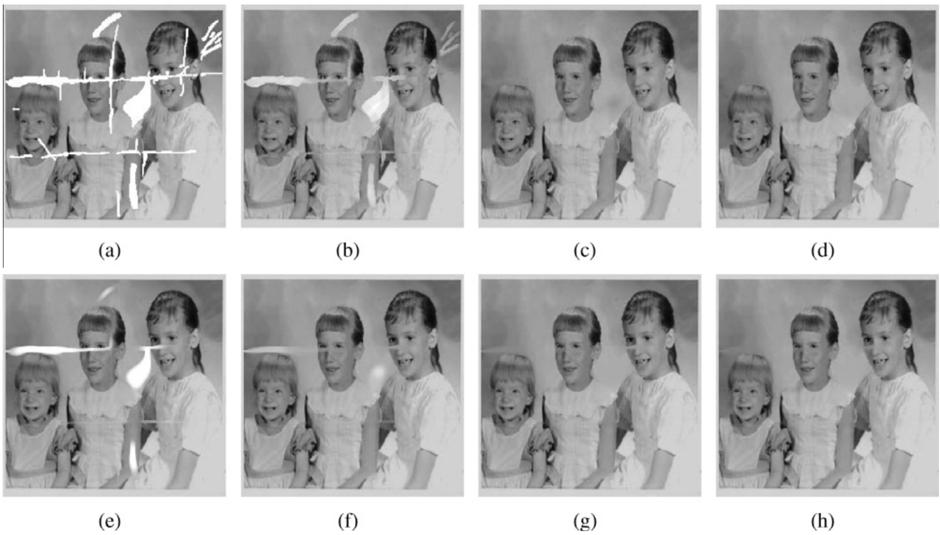



**Fig. 1.** Comparison of gray image inpainting with our algorithm and TV inpainting algorithm in [4]. (a) The test image with inpainting mask (in white), size $256 \times 256$; Results of the proposed algorithm: (b) and (c) intermediate inpainting result at iterations 20 and 100; (d) final inpainted image at iteration 200, computational time = 4.2 s; Results of the TV inpainting algorithm in [4]: (e)–(g) intermediate inpainted image at iteration 100, 500 and 3000; (h) final inpainted image at iteration 5000, computational time = 87.4 s.

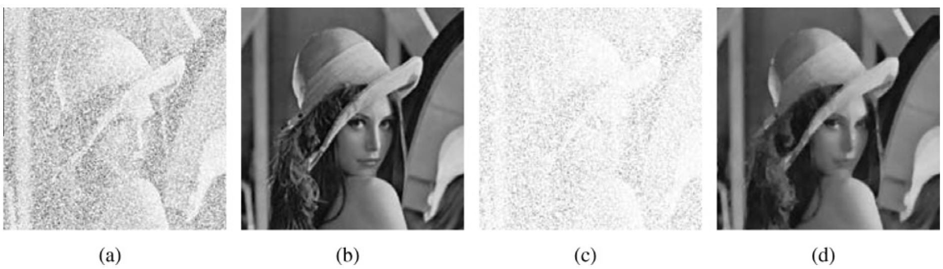



**Fig. 2.** Gray scale image inpainting by the proposed algorithm. (a) Lena image with 30% information left, size $512 \times 512$; (b) inpainted image of (a), PSNR = 32.4 dB: (c) Lena image with 10% information left; (d) inpainted image of (c), PSNR = 25.9 dB.

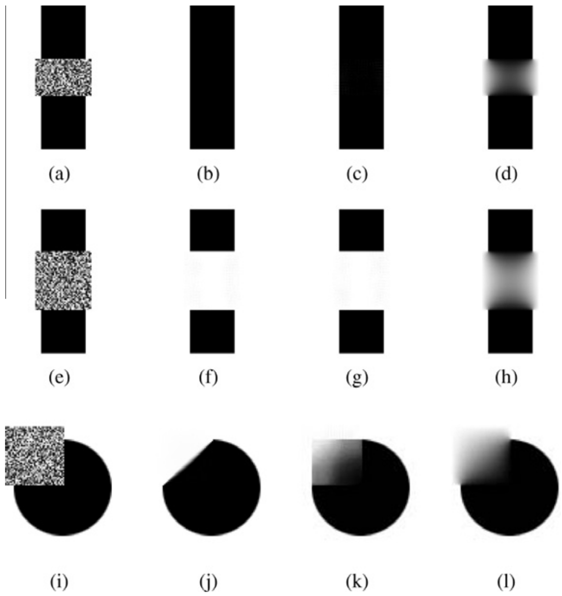


**Fig. 3.** Gray image inpainting by the proposed algorithm and harmonic inpainting. The first column (a), (e), (i) are test images with inpainting mask filled in with random value, with size 100 $\times$ 100; The second column (b), (f), (j) are the inpainting results by our algorithm with isotropic TV norm; The third column (c), (g), (k) are the inpainting results by our algorithm with anisotropic TV norm; The last column (d), (h), (l) are the results of harmonic inpainting.

boundary condition in the inpainting region $D$. Our results are better than those of harmonic inpainting. The inpainting results by our algorithm of Fig. 3a and e are totally different: the bar is connected in Fig. 3b and c while the bar is broken in Fig. 3f and g. Actually, if the height of inpainting mask $h$ is smaller than the width $w$ of the black bar as in Fig. 3a, the inpainting result is a connected bar as Fig. 3b and c, while if $h > w$ as in Fig. 3e, the inpainting result is a broken bar as Fig. 3f and g. This phenomenon is also observed in [4], where the mathematical analysis is given. We also compare the algorithms with isotropic and anisotropic TV norm in the second column and third column. For bar image, the results are the same. While for Fig. 3i, the two TV norms yield totally different results. This phenomenon can be explained by the definition of TV norm.

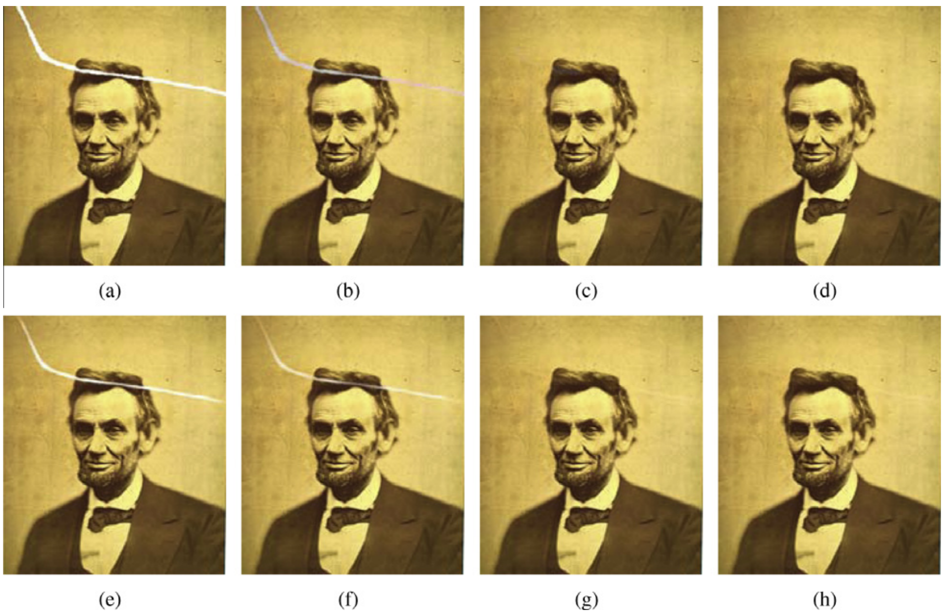
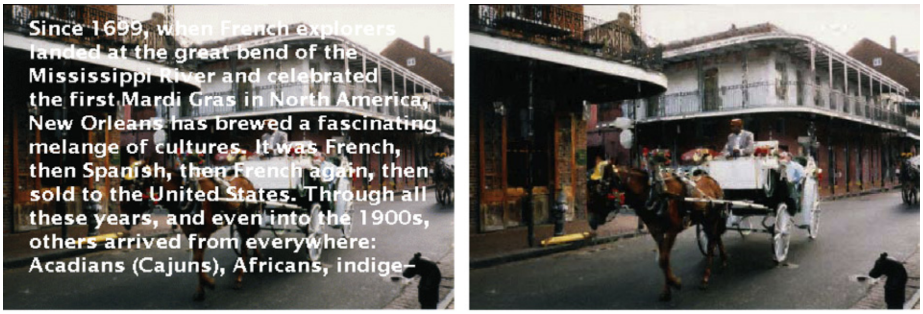


**Fig. 4.** Comparison of color image inpainting with the proposed algorithm and the algorithm in [4]. (a) The test image with mask, size 371 $\times$ 432; Results of the proposed algorithm: (b) and (c) intermediate inpainting result at iterations 20 and 60; (d) final inpainted image at iteration 100, computational time = 14.8 s; Results of model in [4]: (e)–(g) intermediate inpainted image at iteration 100, 200 and 1000; (h) final inpainted image at iteration 1500, computational time = 141.3 s.




(a) (b)

**Fig. 5.** Text removal by our algorithm. (a) Test image with mask, size $438 \times 297$; (b) inpainted image.

(a) (b)

**Fig. 6.** Color image denoising and inpainting by the proposed algorithm. (a) Test image with mask which is contaminated by Gaussian noise with zero mean and standard deviation 20, size $591 \times 394$; (b) denoised and inpainted image by our algorithm.

It is shown in Fig. 3 that TV model has the drawback that it cannot maintain the curvature of isophotes. Remark that the CDD model [7] and Euler elastica model [9] can overcome this drawback by introducing curvature into their models.

### *3.2. Test on color images*

We test our algorithm for color image inpainting in Figs. 4–6. In Fig. 4, we extend the TV inpainting algorithm in [4] to color image and compare our algorithm with it. The intermediate results are displayed. Our algorithm takes 100 iterations and 14.8 s while the implicit algorithm in [4] takes 1500 iterations and 141.3 s to obtain visually comparable results.

In Figs. 5 and 6, two natural color images are tested. Fig. 6a is the test image with mask. It is contaminated by Gaussian white noise with standard deviation 20. Fig. 6b shows that our method can simultaneously denoise and inpaint the image. In this example, the parameters are different from others, $\lambda = 1 * (1 - \chi_D), \tau_1 = 0.015, \tau_2 = 1$. Figs. 5 and 6 show that our algorithm is effective in text removal and scratch removal.

### *3.3. The choice of parameters*

In all the tests the parameters $\lambda, \tau_1, \tau_2$ are chosen by trail and error. We use the same parameters for all the examples except Fig. 6. The reason is in Fig. 6 we consider both inpainting and denoising and the parameters are chosen in order to balance these two parts, while in other tests only inpainting is considered. In general, our algorithm is not very sensitive to these parameters. A small change of these values gives similar results.

## 4. Conclusion

We have proposed a novel algorithm based on operator splitting scheme to solve the classical TV inpainting problem. We have shown that this method is very fast compared with the implicit numerical schemes used early for this problem. First, we calculate the first order optimal condition, i.e. the Euler–Lagrange equation of the energy functional. Usually it is a complicated nonlinear PDE when total variation is involved. Second, we add some new variables to substitute the difficult part in the PDE such that the original PDE can be split into several simple subproblems. Each subproblem has closed form solution. Then the algorithm is obtained by solving these simple subproblems iteratively. The key point of our method depends on suitably choosing variables and operators to split, which is something skillful. On the other hand, we observe in the experiments that our algorithm as well as TV inpainting algorithm in [4] has the drawback that it cannot keep the curvature of isophote (see Fig. 3). To overcome this drawback, geometric information such as curvature must be introduced into the models, for instance, CDD model [7] and Euler elastica model [9]. However, the existing numerical schemes for these models evolve the gradient descent flow, which is very slow due to its time step limitation. In our future work, we plan to extend our algorithm to solve the problem. Since higher order equations or functionals are involved, these problems are very difficult. Moreover, it is hoped that the similar idea can be useful in designing fast and easy to implement numerical schemes to solve other PDEs involved in image processing problems.

## Acknowledgement

F. Li is partially supported by the National Science Foundation of Shanghai (10ZR1410200) and the National Science Foundation of China (11001082, 60773119, 10971066). R. Liu is supported by the Chongqing University of Technology R&D Start Fund (2008ZD18) and the Chongqing CMEC foundation of China (KJ100818) and CQUT Foundation of China (2010ZQ13). J. Fan is supported by the Scientific Research Fund of Zhejiang Provincial Education Department (20060373).

## References

[1] Bertalmío M, Sapiro G, Caselles V, Ballester C. Image inpainting. Computer Graphics, SIGGRAPH 2000, p. 417–24.
[2] Ballester C, Bertalmío M, Caselles V, Sapiro G, Verdera J. Filling-in by joint interpolation of vector fields and gray levels. IEEE Trans Image Process 2001;10:1200–11.
[3] Bertalmío M, Bertozzi A, Sapiro G. Navier–Stokes, fluid–dynamics and image and video inpainting. In: Proceedings of the 2001 IEEE computer society conference on computer vision and pattern recognition (CVPR 2001), vol. 1, 2001. p. 355–62.
[4] Chan TF, Shen J. Mathematical models for local nontexture inpaintings. SIAM J Appl Math 2002;62:1019–43.
[5] Rudin L, Osher S, Fatemi E. Nonlinear total variation based noise removal algorithms. Phys D 1992;60:259–68.
[6] Chan TF, Kang SH. Error analysis for image inpainting. J Math Imag Vis 2006;26:85–103.
[7] Chan TF, Shen J. Non-texture inpainting by curvature-driven diffusion (CDD). J Vis Comm Image Represent 2001;12:436–49.
[8] Masnou S, Morel JM. Level lines based disocclusion. In: Proceedings of IEEE international conference on image processing 1998 (ICIP 98), vol. 3, p. 259–63.
[9] Chan TF, Kang SH, Shen J. Euler's elastica and curvature based inpainting. SIAM J Appl Math 2002;63:564–92.
[10] Grossauer H, Scherzer O. Using the complex Ginzburg–Landau equation for digital inpainting in 2D and 3D. In: Proceedings of 4th international conference, scale space 2003, LNCS, vol. 2695, p. 225–36.
[11] Tai XC, Osher S, Holm R. Image inpainting using a TV-Stokes equation. Image Processing based on partial differential equations. Heidelberg: Springer; 2006.
[12] Criminisi A, Perez P, Toyama K. Region filling and object removal by exemplar-based image inpainting. IEEE Trans Image Process 2004;13:1200–12.
[13] Patwardhan KA, Sapiro G, Bertalmío M. Video inpainting of occluded and occluding objects. In: Proceedings of IEEE international conference on image processing 2005 (ICIP 2005), vol. 2, p. 69–72.
[14] Ranjbar M, Kasaei S. Spatial error concealment: a novel exemplar-based approach using segmentation. Comput Elect Eng 2009;35:536–48.
[15] Grossauer H. A combined PDE and texture synthesis approach to inpainting. In: Proceedings of eighth European conference on computer vision (ECCV 2004), LNCS, vol. 3022.
[16] Bertalmío M, Vese L, Sapiro G, Osher S. Simultaneous structure and texture image inpainting. IEEE Trans Image Process 2003;12:882–9.
[17] Arias P, Caselles V, Sapiro G. A variational framework for non-local image inpainting. In: Seventh international conference of energy minimization methods in computer vision and pattern recognition (EMMCVPR 2009), LNCS, vol. 5681. p. 345–58.
[18] Yau AC, Tai XC, Ng MK. $L_0$-Norm and total variation for wavelet inpainting. In: Second international conference on scale space and variational methods in computer vision (SSVM 2009), LNCS, vol. 5567, p. 539–51.
[19] Cai JF, Chan RH, Shen L, Shen Z. Simultaneously inpainting in image and transformed domains. Numer Math 2009;112:509–33.
[20] Shen CM. Geometric variational principle based image processing method. Ph.D. thesis of East China Normal Univ. (in Chinese), 2009.
[21] Lions PL, Mercier B. Splitting algorithms for the sum of two nonlinear operators. SIAM J Numer Anal 1979;16:964–79.
[22] Ma S, Yin W, Zhang Y, Chakraborty A. An efficient algorithm for compressed MR imaging using total variation and wavelets. In: Proceedings of the IEEE conference on computer vision and pattern recognition 2008 (CVPR 2008). p. 1–8.
[23] Chambolle A. An algorithm for total variation minimization and applications. J Math Imaging Vis 2004;20:89–97.
[24] Goldstein T, Osher S. The split Bregman method for L1 regularized problems. SIAM J Imaging Sci 2009;2:323–43.
[25] Vese L, Osher S. Numerical methods for p-harmonic flows and applications to image processing. SIAM J Numer Anal 2002;40:2085–104.

**Fang Li** received her M.Sc. degree in Mathematics from the South West China Normal University in 2004 and received her Ph.D. degree in Mathematics from the East China Normal University (ECNU) in 2007. She is currently a lecturer in the Department of Mathematics, ECNU. Her research interests include anisotropic diffusion filtering, variational methods and PDEs in image processing.

**Chaomin Shen** is a lecturer of computer science at ECNU. He received his Ph.D. and Master degrees from ECNU and the National University of Singapore (NUS), respectively, all in Mathematics. His research interest is image processing using mathematical methods.

**Ruihua Liu** received his Ph.D. degree in Mathematics from ECNU in 2008. He is currently a lecturer in the Department of Mathematics and Statistics, Chongqing University of Technology. His research interests are blind image deblurring, super-resolution image reconstruction and the variational methods in image processing.

**Jinsong Fan** received his M.Sc. degree in mathematics from ECNU in 1996 and received his Ph.D. degree in Mathematics from ECNU in 2007. Currently, he is with the College of Mathematics and Information Science at Wenzhou University. His research interests include image processing and pattern recognition.